Nonlinear Dynamics and Systems Theory, **25** (6) (2025) 656–668

Global Theoretical Investigation of Diffusion Driven Instability for Three Coupled Equations of a Reaction Diffusion System

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Received: July 5, 2025; Revised: November 20, 2025

Abstract: In this paper, we investigate the mechanism called DDI (Diffusion Driven Instability) for a full three dimensional matrix of diffusion coefficients. We apply a linear approach in the neighborhood of an arbitrary equilibrium point using the Routh-Hurwitz stability criterion and we study the existence of at least one eigenvalue with positive real part of the matrix $A(k)$. Our main result is the proof of sufficient and necessary condition for the Turing instability. The research is extended to a reaction-diffusion system for three species.

Keywords: *reaction-diffusion system; Turing instability; cross diffusion; predator-prey.*

Mathematics Subject Classification (2020): 35K57, 35B36, 35B32, 92D25, 92D40, 37N25, 70K42.

1 Introduction

Back in the 1950s, Alan Turing published a paper under the title "The Chemical Basis of Morphogenesis". Turing demonstrated that under certain circumstances, chemicals can react and diffuse in a way that results in solutions that do not have concentration equilibrium. To study the process of morphogenesis, he took into account two coupled reaction-diffusion systems. Mathematically, Turing's idea was as follows:

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1 \Delta u + f(u,v), & t > 0 \quad x \in \Omega, \\ \dfrac{\partial v}{\partial t} = d_2 \Delta v + g(u,v), & t > 0 \quad x \in \Omega, \end{cases} \tag{E}$$

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