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Two-Parameter Quasi-Boundary Regularization for Backward Cauchy Problems

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Abstract: We propose a two-parameter quasi-boundary regularization method for solving the ill-posed backward Cauchy problem. The Lambert W function is employed for the first time to derive enhanced stability bounds and refined convergence rates. Our approach perturbs the final data via two distinct parameters, providing better control of approximation errors. We prove the regularised problem's well-posedness and derive novel Hölder-Lambert stability estimates. Numerical experiments confirm that our method improves the accuracy of estimated errors, especially under high noise levels.

Keywords: *ill-posed problems; regularization; quasi-boundary value method; backward parabolic problem; stability analysis; Hölder–Lambert stability.*

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1 Introduction

Let H be a Hilbert space with the inner product $(.,.)$ and the norm $\|.\|$, and A be a self-adjoint operator on H. Assume that A admits an orthonormal eigenbasis $(\varphi_i)_{i\geq 1}$ in H, associated to the eigenvalues $(\lambda_i)_{i\geq 1}$ such that

$$0 < \lambda_1 < \lambda_2 < ...\text{et} \lim_{i\longrightarrow+\infty} \lambda_i = +\infty.$$

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