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Fokker-Planck Equation and Its Application in Production Function

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Abstract: The one-dimensional Fokker-Planck equation (FPE) with drift and diffusion coefficients depending on the space variable is identified by a semigroup approach. The stationary solution u_s of the FPE induces a Hilbert space X, i.e., $L^2(a,b)$ with an inner product weighted by u_s. The backward Fokker-Planck operator A generates a C_0-semigroup in X. The well-posedness for the FPE follows the well-posedness for the Cauchy problem generated by A. The solution u is asymptotically stable with respect to u_s as $t \to \infty$. Furthermore, if the ratio of the drift to diffusion coefficients is nondecreasing, then u is a nonnegative classical solution. As an application, the backward Fokker-Planck operator A confirms the well-posedness for production function equations. In case $X = L^2(0,\infty)$, the operator A has a continuous spectrum generating the Gaus-Weierstrass semigroup.

Keywords: *Fokker-Planck equation; stationary solution; C_0-semigroup; well-posed; production function.*

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1 Introduction

The Fokker-Planck equation (FPE) which originally describes a Brownian motion of a particle has wide applications leading to many interdisciplinary studies, for example, in solid state physics, quantum optics, chemical physics, theoretical biology, circuit theory, plasma waves, finance and economics. Concretely, the applications of the FPEs are found in differential equations and stochastic processes [1, 2], bilinear control systems

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