Nonlinear Dynamics and Systems Theory, **25** (6) (2025) 585–595

Solvability of Functional Equations' Classes Arising in Dynamic Programming Using Fixed-Point Technique

Ahlem Achichi [1], Iqbal M. Batiha [2,3,*], Leila Benaoua [1], Taki-Eddine Oussaeif [1], Nidal Anakira [4] and Tala Sasa [5]

[1] *Department of Mathematics and Informatics, Dynamic and Control Systems Laboratory, Oum El Bouaghi University, Algeria.*
[2] *Department of Mathematics, Al Zaytoonah University of Jordan, Amman 11733, Jordan.*
[3] *Nonlinear Dynamics Research Center (NDRC), Ajman University, Ajman 346, UAE.*
[4] *Faculty of Education and Arts, Sohar University, Sohar 3111, Oman.*
[5] *Applied Science Research Center, Applied Science Private University, Amman, Jordan.*

Received: February 6, 2025; Revised: November 2, 2025

Abstract: This paper is devoted to the existence, uniqueness, and iterative approximation of solutions for the classes of functional equations arising in dynamic programming of multistage decision processes. The findings presented here extend and integrate several results from the existing literature. Illustrative examples are also provided to emphasize the significance of the main results. The approach is based on fixed point techniques applied in suitable function spaces. Furthermore, our results unify a variety of known theorems within a broader and more flexible framework.

Keywords: *functional equations; dynamic programming; fixed point; iterative approximation.*

Mathematics Subject Classification (2020): 70K20, 70K30, 93C10.

1 Introduction

In 1922, Banach [1] proved his celebrated fixed point theorem, commonly known as the Banach contraction principle. Bellman [2, 3] introduced and explored the existence of solutions for a class of functional equations arising in dynamic programming. Since then, many researchers (see [4–11]) have studied the existence and uniqueness of solutions to functional equations by modifying the conditions of Bellman's equations in the context of multistage decision processes.

* Corresponding author: mailto:i.batiha@zuj.edu.jo