



## On Existence and Uniqueness of Solution of Heat Equations in Quasi-Metric Spaces

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**Abstract:** In this paper, we prove the existence and uniqueness of solutions to heat equations in quasi-metric spaces by applying the  $\phi G$ -contraction in this setting. This type of contraction is analogous to the  $\psi F$ -contraction introduced by Secelean et al. in 2019. In the  $\psi F$ -contraction, we have  $F : \mathbb{R}^+ \rightarrow \mathbb{R}$  as an increasing mapping and  $\psi : (-\infty, \mu) \rightarrow \mathbb{R}$  for some  $\mu \in \mathbb{R}^+ \cup \{\infty\}$  as an increasing and continuous function such that  $\psi(t) < t$  for every  $t$  in  $(-\infty, \mu)$ . Meanwhile, in the  $\phi G$ -contraction, we have  $G$  as a strictly increasing mapping from  $\mathbb{R}^+ \cup \{0\}$  to  $\mathbb{R}^+ \cup \{0\}$ . Also  $\phi : (-\infty, \mu) \rightarrow \mathbb{R}^+ \cup \{0\}$  as a strictly increasing and continuous function satisfying  $\phi(t) < t$  for all  $t$  and  $\phi(0) = 0$ . This approach also provides a framework for solving nonlinear equations.

**Keywords:** *fixed point theories; heat equations; quasi-metric spaces.*

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### 1 Introduction

In the last century, fixed points have become an important topic in pure and applied mathematics, as well as in nonlinear dynamics, see, for example, [1–6]. The concept of fixed points and their associated mappings is crucial in investigating the existence and uniqueness of solutions to various mathematical models. The study of fixed points began with the Banach Contraction Principle in complete metric spaces in 1922 (see [7]). Afterward, many researchers introduced other types of fixed points in complete metric spaces and found their applications both in pure and applied mathematics.

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