



Limit Cycles for a Class of Generalized Liénard Polynomial Differential Systems via the First-Order Averaging Method

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Abstract: In this paper, using the averaging method of first order, we compute the maximum number of limit cycles that can bifurcate from the periodic orbits of the center $\dot{x} = -y^{2p-1}$, $\dot{y} = x^{2q-1}$ with p and q being positive integers, under perturbation in the particular class of the generalized Liénard polynomial differential systems.

Keywords: *limit cycle; averaging method; periodic orbit; polynomial differential system.*

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1 Introduction

The second part of the Hilbert's 16th problem [9, 17] aims to find a uniform upper bound for the number of limit cycles of all polynomial differential systems of a given degree. The limit cycles problem and the center problem are concentrated on specific classes of systems. For instance, much has been written on Kukles systems, Duffing systems, Mathieu differential equations, Kolmogorov systems (see for example, [5, 10, 11, 15]) and Liénard systems, that is, systems of the form

$$\dot{x} = y, \quad \dot{y} = -x - f(x),$$

where $f(x)$ is a polynomial in the variable x of degree m . The motivation in the Liénard family is because it is one of the most important families related to the Hilbert's

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