



The Duffing Oscillator: Metamorphoses of 1 : 2 Resonance and Its Interaction with the Primary Resonance

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Abstract: We investigate the 1 : 2 resonance in the periodically forced asymmetric Duffing oscillator, it is created via the period-doubling of the primary 1 : 1 resonance or forming independently and coexisting with the primary resonance. We compute the steady-state asymptotic solution – the amplitude-frequency response function. Working in the framework of differential properties of implicit functions, we discover and describe complicated metamorphoses of the 1 : 2 resonance and its interaction with the primary resonance.

Keywords: *Duffing equation; resonances; singularities; metamorphoses.*

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1 Introduction and Motivation

A period-doubling cascade of bifurcations is a typical route to chaos in nonlinear dynamical systems. A generic example is the asymmetric Duffing oscillator governed by the non-dimensional equation

$$\ddot{y} + 2\zeta\dot{y} + \gamma y^3 = F_0 + F \cos(\Omega t), \quad (1)$$

which has a single equilibrium and a one-well potential [1], where ζ , γ , F_0 , F are parameters and Ω is the angular frequency of the periodic force.

Szemplińska-Stupnicka elucidated the period-doubling scenario in the dynamical system (1) in a series of far-reaching papers [2–4]; see also [1] for a review and further results.

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