



Double-Phase System with Neumann Boundary Condition

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Abstract: This paper investigates the existence of multiple solutions for double-phase systems subject to Neumann conditions. The study is conducted within the framework of Sobolev spaces featuring variable exponents. Assuming appropriate conditions on the given data, we establish the existence of at least two weak solutions, each characterized by distinct energy signs. We employ the Nehari manifold and variational method as the foundation for our approach.

Keywords: *Neumann boundary; two-phase operator; weak solutions; convex-concave source.*

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1 Introduction

Let a bounded domain $\mathcal{U} \subseteq \mathbb{R}^N$, $N \geq 2$, with the Lipschitz boundary $\partial\mathcal{U}$ be given. Consider the following double-phase system:

$$\left\{ \begin{array}{ll} \mathcal{L}_{p(y),q(y)}^{\mu_1(y)} u = \lambda_1 |u(y)|^{q(y)-2} u(y) + \frac{2r(y)}{r(y)+s(y)} |u(y)|^{r(y)-2} u(y) |v(y)|^{s(y)} & \text{in } \mathcal{U}, \\ \mathcal{L}_{p(y),q(y)}^{\mu_2(y)} v = \lambda_2 |v(y)|^{q(y)-2} v(y) + \frac{2s(y)}{r(y)+s(y)} |u(y)|^{r(y)} |v(y)|^{s(y)-2} v(y) & \text{in } \mathcal{U}, \\ (Du(y) |^{p(y)-2} Du + \mu_1(y) | Du(y) |^{q(y)-2} Du) \cdot \nu = h_1(y, u(y)) & \text{on } \partial\mathcal{U}, \\ (|Dv(y) |^{p(y)-2} Dv + \mu_2(y) | Dv(y) |^{q(y)-2} Dv) \cdot \nu = h_2(y, v(y)) & \text{on } \partial\mathcal{U}, \end{array} \right. \quad (1)$$

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