



Resolvability of Boundary Value Problems with Integral Conditions and Solution's Approximation by Homotopy Perturbation Method

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Abstract: The Telegraph problem is a linear time-partial differential equation that models the transmission of electrical impulses through a cable. It consists of two coupled partial differential equations that describe the voltage and current within the cable, capturing both wave-like propagation and diffusive effects. This equation integrates elements from the wave equation and the heat equation to account for resistive losses and signal propagation speed. The objective of this study is to investigate the existence and uniqueness of a strong solution to the Telegraph problem under purely integral conditions. The analysis is conducted using the operator density method derived from the problem framework and the energy inequality approach. To approximate the desired solution, a combination of the Laplace transform technique and the homotopy perturbation method is employed. This approach yields solutions in the form of rapidly convergent series, and the convergence of these series is rigorously established. The findings indicate that the proposed methodology is highly effective and applicable to a broad class of mathematical problems. To validate these results, several illustrative examples are provided, demonstrating the accuracy of the proposed method by comparing approximate solutions with exact solutions.

Keywords: *telegraph equation; purely integral conditions; a priori estimate; Laplace transform; homotopy perturbation method; Stehfest algorithm.*

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