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Existence Results for a Class of Hybrid Fractional Differential Equations Involving Generalized Riemann-Liouville Fractional Derivatives

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Abstract: This research paper deals with the uniqueness of solutions for a second-type hybrid fractional differential equation that involves generalized Riemann-Liouville fractional derivatives using the Banach contraction principle. We also discover at least one solution by employing certain assumptions and the Schaefer fixed point theorem. Subsequently, the Ulam–Hyers stability is discussed. Finally, we enhance our study with a relevant example.

Keywords: *hybrid fractional differential equations, generalized Riemann-Liouville fractional derivatives, existence and uniqueness of solution, Ulam-Hyers stability.*

Mathematics Subject Classification (2020): 34A12, 34K37, 93D05, 70k20.

1 Introduction

Fractional differential equations (FDEs) are a fascinating area of mathematics dealing with derivatives of non-integer order and allowing for a more nuanced description of systems with memory effects or long-range interactions. Solving FDEs can be challenging due to the non-integer order of the derivatives, requiring specialized techniques such as fractional calculus. In general, fractional differential equations provide a powerful tool for understanding complex systems with given dynamics [1, 6, 7, 9]. Indeed, though the operations of FDEs are relatively broad, they can not be applied to all systems. The researchers have shown that certain phenomena related to material heterogeneity cannot be adequately modeled using fractional derivatives. In view of this fact, a solution to this

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