Nonlinear Dynamics and Systems Theory, **25** (4) (2025) 424–433

Bright and Dark Solitons via Homoclinic Dynamics in Helmholtz-Type DNLS Equations

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Received: October 29, 2024; Revised: July 13, 2025

Abstract: The existence of homoclinic orbits in a dynamical system has interesting consequences for its behavior. This is the case in this paper, where we present a model of the discrete nonlinear Schrödinger equation under the Helmholtz operator. We give the fundamental theorem of the existence of a homoclinic (heteroclinic) orbit for a particular class of reversible planar maps. Homoclinic structures are known to be sources of sensitivity that, under small perturbations, can bifurcate solutions. The problem of the existence of solitons has therefore been replaced by that of the existence of homoclinic solutions. We prove the existence of bright and dark solitons in a certain case of nonlinearity.

Keywords: *discrete Schrödinger equation; Helmholtz operator; homoclinic orbits; heteroclinic orbits; reversible planar maps.*

Mathematics Subject Classification (2020): 35Q55, 35Q51, 37K60, 70K44, 93-02.

1 Introduction

Over the last decade, the existence of discrete solitons in DNLS equations has become a hot topic of many studies, to mention just a few, refer to [7, 11–13, 15–17]. These include variational methods, central manifold reduction, and the Nehari manifold approach. A good number of these papers take into account DNLS equations with constant coefficients, and their conclusions have been presented in [7, 12, 15, 16, 19]. DNLS equations with periodic coefficients have recently appeared in the physics literature, and this phenomenon can be identified by numerical simulations [11, 13].

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