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The Limit-Point/Limit-Circle Problem for Fractional Differential Equations

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Abstract: In this paper, the author examines the famous limit-point/limit-circle problem introduced by Hermann Weyl more than one-hundred years ago (1910) and popularized in Volume 2 of the well known treatise by Dunford and Schwartz. They visit this problem in the case where it involves fractional derivatives; this has not been studied before.

Keywords: *fractional equation; limit-point problem; limit-circle problem; square integrability.*

Mathematics Subject Classification (2020): 34B20, 34C10.

Dedication: This paper is dedicated to the memory of T. A. (Ted) Burton on the occasion of the ninetieth anniversary of his birth.

1 Introduction

A problem with more than a one-hundred year history going back to the seminal work of Hermann Weyl in [27] is the limit-point/limit-circle problem. It began with his work on eigenvalue problems for the second order linear differential equation

$$(a(t)y')' + r(t)y = \lambda y, \quad t \in [0, \infty), \quad \lambda \in \mathbb{C}, \tag{C}$$

which he classified as being of the *limit-circle* type if every solution is square integrable (belongs to L^2), and to be of *limit-point* type if at least one solution does not belong to L^2. This problem has important connections to the solution of certain boundary value problems as can be seen in the works of Titchmarsh [25, 26].

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