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Stability Analysis of a COVID-19 SIR Model with Direct and Indirect Transmission

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Abstract: This paper develops a SIR model for COVID-19 that incorporates both direct and indirect transmission dynamics through two distinct incidence rates. To capture the infection rate, we employ a nonlinear Beddington-DeAngelis function and a bilinear incidence function. The model's solutions are shown to be positive and bounded, with two equilibrium points identified: the disease-free equilibrium E_0 and the endemic equilibrium E^*. We establish that E_0 is locally and globally asymptotically stable when the basic reproduction number $R_0 < 1$. Conversely, under specific parameter conditions, E^* is uniformly asymptotically stable for $R_0 > 1$. Numerical simulations are provided to validate the theoretical results.

Keywords: *epidemic model; direct-indirect transmission; incidence function; stability analysis.*

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1 Introduction

The COVID-19 pandemic has spurred research across many fields, including the development of mathematical models to assess the impact of interventions on disease control. Kermack and McKendrick [10] pioneered the use of compartmental models for disease dynamics research, leading to the development of various models such as SIR, SIRS, and SEIRS [1, 2, 7, 9, 11, 12].

Incidence functions are crucial in epidemic models as they determine how susceptible individuals transition to infected, significantly influencing model predictions. Epidemiological models often assume well-mixed populations in uniform environments. These models typically use the bilinear incidence rate βSI [10,17] or the standard incidence rate

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