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Dynamics of a Fractional Order Model for Mycobacterium Tuberculosis with Caputo-Fabrizio Derivatives

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Abstract: Tuberculosis is an infectious disease caused by the Mycobacterium tuberculosis (Mtb), bacteria that primarily attacks the lungs. Currently, tuberculosis remains a major public health challenge. This study develops a fractional-order mathematical model using the Caputo-Fabrizio derivatives to explore the growth dynamics of MTb in relation to vaccine administration. The methodology consists of the following steps: model formulation, equilibrium point determination, computation of the fundamental reproduction number R_0, equilibrium point stability analysis, and numerical simulation utilizing the Adam-Bashforth 3-step method. The main result reveals that the nonlinear dynamics of the model exhibits significant sensitivity to the fractional order. The model indicates that the infection-free equilibrium point is locally asymptotically stable if $R_0 < 1$, and the endemic equilibrium point is also locally asymptotically stable under specific circumstances. The fractional order can greatly influence the convergence rate towards equilibrium points, as numerical simulations further highlight that smaller fractional orders accelerate the convergence of immune response cells to stability, demonstrating the potential of fractional calculus to capture complex biological dynamics more effectively.

Keywords: *mycobacterium tuberculosis; fractional-order model; Caputo-Fabrizio derivatives; numerical simulation.*

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