Blow up of Nonlinear Hyperbolic Equation with Variable Damping and Source Terms

S. Abdelhadi * and I. Hamchi

Laboratory of PDE and Applications, Department of Mathematics, University of Batna 2, Algeria.

Abstract: In this work, we consider a nonlinear hyperbolic equation with variable damping and source terms. Our aim is to prove that the solution with negative initial energy blows up in finite time.

Keywords: *hyperbolic equation; damping term; source term; variable exponents; blow up.*

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1 Introduction

In this work, we consider the following problem

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\begin{cases}
u_{tt} - div\,(A\nabla u) + u_t\,|u_t|^{m(.)-2} = u\,|u|^{p(.)-2} & \text{in } \Omega \times (0,T)\,, \\
u = 0 \quad \text{on } \partial\Omega \times (0,T)\,, & (P)\\
u(0) = u_0 \quad \text{and} \quad u_t(0) = u_1 \quad \text{in } \Omega,
\end{cases}
$$

where $T > 0$, Ω is a bounded domain of $\mathbb{R}^n$ ($n \in \mathbb{N}^*$) with a smooth boundary $\partial\Omega$. $A = A(x,t)$ is an $n \times n$ symmetric matrix with real coefficients. The exponents $m(.)$ and $p(.)$ are given measurable functions on Ω.

When $A =$ Identity, the bibliography of works concerning problems of existence and nonexistence of global solution is truly long. **In the case of constant damping and source terms,** Ball [3] in 1977, considered the wave equation with source term and proved the blow up of solution when the energy of the initial data is negative. Haraux and Zuazua [8] in 1988, proved that the damping term of polynomial or arbitrary growth

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* Corresponding author: `mailto:s.abdelhadi@univ-batna2.dz`