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Exponential Decay of Timoshenko System with Fractional Delays and Source Terms

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Abstract: The objective of this paper is to analyse the asymptotic behavior of a Timoshenko beam system with fractional delays and nonlinear external sources. Under suitable conditions on the damping, delay and initial data, we obtain exponential decay rate of the solution.

Keywords: *Timoshenko system; energy decay; nonlinear systems; fractional delay.*

Mathematics Subject Classification (2020): 35B40, 47D03, 74D05, 93D23, 93D15.

1 Introduction

In this work, we study the following Timoshenko system with fractional delays:

$$\begin{cases} \rho_1\varphi_{tt} - k\left(\varphi_x+\psi\right)_x + a_1\partial_t^{\alpha,\beta}\varphi\left(t-s\right) + \alpha_1\varphi_t = |\varphi|^{p-2}\varphi, \\ \rho_2\psi_{tt} - b\psi_{xx} + k\left(\varphi_x+\psi\right) + a_2\partial_t^{\alpha,\beta}\psi\left(t-s\right) + \alpha_2\psi_t = |\psi|^{q-2}\psi, \\ \varphi(x=0,t) = \psi(x=0,t) = \varphi(x=L,t) = \psi(x=L,t) = 0, \\ \varphi(x,t=0) = \varphi_0(x),\ \psi(x,t=0) = \psi_0(x), \\ \varphi_t(x,t=0) = \varphi_1(x),\ \psi_t\left(x,0\right) = \psi_1(x), \\ \varphi_t\left(x,t-s\right) = f_0\left(x,t-s\right), t\in(0,s), \\ \psi_t\left(x,t-s\right) = g_0\left(x,t-s\right), t\in(0,s), \end{cases} \tag{1}$$

where $x\in\Omega=(0,L), L>0, t\in\mathbb{R}_+^\star, \rho_1,\rho_2,a_1,a_2,\alpha_1,\alpha_2,b$ and k are positive real constants. The constant $s>0$ is the time delay and the exponents p and q satisfy $p>2$

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