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A Note on Linear Matrix Functions and Applications

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Abstract: This paper focuses on some algebraic characterizations between linear matrix functions (LMFs) and their domains defined over the field of complex numbers $\mathbb{C}$. We discuss the intersection, as well as the inclusion of two domains of some LMFs. By applying specific algebraic methods on ranks and ranges, we consider certain forms of LMFs, where the general solutions can be expressed via specific explicit LMFs to establish some relationships between their domains. As a consequence, we have obtained a well-known result of Lin and Wang.

Keywords: *linear matrix function; algebraic method; generalized inverse; general solutions; rank.*

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1 Introduction

In this work, we use the notation $\mathbb{C}^{n\times m}$ to represent the set of all $n \times m$ complex matrices. The symbols A^*, $\mathfrak{R}(A)$, $r(A)$ and I_n denote the conjugate transpose, the range, the rank of the matrix A and the identity matrix of order n, respectively. The Moore-Penrose inverse of a matrix $A \in \mathbb{C}^{n\times m}$ is defined as the unique $m \times n$ complex matrix denoted by A^+ satisfying the following four equations:

$$AA^+A = A,\ \ A^+AA^+ = A^+,\ \ \left(AA^+\right)^* = AA^+,\ \ \left(A^+A\right)^* = A^+A. \tag{1}$$

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