Analysis and Numerical Approximation of the Variable-Order Time-Fractional Equation

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Abstract: In this paper, we investigate a fully implicit finite scheme approximation equation (IFSAE) of the 1-D linear variable-order time-fractional diffusion equation (VOTFDE). The numerical method of solving differential equations by approximating them with difference equations is called the implicit finite difference method (IFDM). The first-order numerical scheme, stability, consistency and convergence of the method are proven. Moreover, the scheme is implemented on two test problems and some graphical results are offered to verify the theoretical analysis of the above scheme and illustrate the effectiveness of the suggested schemes.

Keywords: *fractional derivatives; discretization; implicit numerical scheme; stability; convergence.*

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1 Introduction

Applied mathematics is the application of mathematical methods in various fields such as physics, engineering, medicine, biology, finance, economics, computer science and industry. Thus, applied mathematics is a combination of mathematics and engineering. Operational calculus, also called operational analysis, is a technique used to transform analytical problems, especially differential equations, into algebraic problems, usually the problem of solving a polynomial equation. Numerical analysis is the study of algorithms that use numerical approximation (as opposed to symbolic manipulations) to the problems of mathematical analysis (as distinct from discrete mathematics). This is the study

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