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The Twin-Well Duffing Equation: Escape Phenomena, Bistability, Jumps, and Other Bifurcations

J. Kyzioł and A. Okniński *

Politechnika Świętokrzyska, Al. 1000–lecia PP 7, 25-314 Kielce, Poland.

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Abstract: In this work, we investigate the escape phenomenon, the onset of bistability, jumps, as well as other bifurcations present in the twin-well Duffing equation. Based on the known steady-state asymptotic solution – the amplitude-frequency implicit function – and using the theory of differential properties of implicit functions, we compute the singular and critical points of this function. This enables us to predict several bifurcations present in the dynamical system under study. The main result is the calculation of the escape bifurcation set – the set of parameters for which escape phenomena occur, and equations to compute the onset of bistability.

Keywords: *metamorphoses of amplitude-frequency curves; bifurcations.*

Mathematics Subject Classification (2010): 34C05, 34C25, 34E05, 37G35, 70K20, 70K30, 70K50.

1 Introduction and Motivation

We study the dynamics of a vibrating system governed by the Duffing equation with negative linear stiffness [1, 2]:

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} - \alpha x + \gamma x^3 = f\cos\omega t \qquad (k,\ \alpha,\ \gamma > 0)\,. \tag{1}$$

After rescaling the variables as in [2]:

$$\tau = t\sqrt{\alpha},\ y = \alpha x,\ h = k/\sqrt{\alpha} > 0,\ c = \gamma/\alpha^3 > 0,\ \Omega = \omega/\sqrt{\alpha}, \tag{2}$$

we obtain the nondimensional Duffing equation

* Corresponding author: mailto:fizao@tu.kielce.pl