Class of Nilpotent Distributions and $\mathfrak{N}_2$-Distributions

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Abstract: This paper presents a sufficient condition for two vector fields X and Y to have the squares noncommutative, i.e., $[X^2, Y^2] \neq 0$, in the case when X and Y span a 3-nilpotent distribution. And when the nilpotent disributions of class 2 or 3 are spanned from more than two vector fields, it gives the same result.

1 Introduction

The theory of subelliptic operators plays an important role in many applications in nonlinear dynamics and system theory, robotics and mechanical systems, optimal control of nonlinear systems, see [1, 9].

The subelliptic operator is a particular of case hypoelliptic differential equations. Hypoelliptic equations involve operators that are neither purely elliptic (like Laplace's equation) nor hyperbolic (like the wave equation), but rather fall in between. These equations often arise in the context of modeling systems with varying degrees of regularity and smoothness.

An example is the study of heat conduction in materials with varying degrees of conductivity, with a heat diffusion being non-uniform in all directions. The equation $\frac{\partial}{\partial t} - L\,u = 0$ gives a more efficient description in that direction, where $u(x,t)$ is the heat kernel and the subelliptic operator L is defined in the differential manifold. And the

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