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# Maximum and Anti-Maximum Principles for Boundary Value Problems for Ordinary Differential Equations in Neighborhoods of Simple Eigenvalues

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**Abstract:** It has been shown that, under suitable hypotheses, for boundary value problems of the form $Ly + \lambda y = f$, $BCy = 0$, where $L$ is a linear differentiable operator and $BC$ denotes the linear boundary operator, there exists $\Lambda > 0$ such that $f \geq 0$ implies $\lambda y \geq 0$ for $\lambda \in [-\Lambda, \Lambda] \setminus \{0\}$, where $y$ is the unique solution of $Ly+\lambda y = f$, $BCy = 0$. So, the boundary value problem satisfies a maximum principle for $\lambda \in [-\Lambda, 0)$ and the boundary value problem satisfies an anti-maximum principle if $\lambda \in (0, \Lambda]$. Moreover, this information is provided in the one inequality, $\lambda y \geq 0$. In this study, we shall provide suitable hypotheses such that for boundary value problems of the form $Ly+\beta y' = f$, $BCy = 0$, where $L$ is an ordinary differentiable operator and $BC$ denotes the boundary operator, there exists $\mathcal{B} > 0$ such that $f \geq 0$ implies $\beta y' \geq 0$ for $\beta \in [-\mathcal{B}, \mathcal{B}] \setminus \{0\}$, where $y$ is the unique solution of $Ly+\beta y' = f$, $BCy = 0$. Under suitable boundary conditions, one obtains sign properties on solutions and derivatives of solutions. Two examples satisfying the suitable hypotheses are provided and one application of monotone methods is provided to illustrate an application of the main result.

**Keywords:** *maximum principle; anti-maximum principle; ordinary differential equation; boundary value problem.*

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