Nonlinear Dynamics and Systems Theory, **23** (5) (2023) 475–486

An Algorithm for Solving First-Kind Two-Dimensional Volterra Integral Equations Using Collocation Method

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Received: July 2, 2023; Revised: November 9, 2023

Abstract: The proposed study presents a collocation method to address two types of two-dimensional Volterra integral equations (2D VIEs): nonlinear first kind and linear second kind. The nonlinear equations of the first kind are transformed into the linear second kind equations. A convergent algorithm using the Taylor polynomials is developed to construct a collocation solution that approximates the solution of 2D VIEs of the second kind. The study includes various numerical examples to compare the results of different methods and demonstrate the proposed approach's accuracy and validity. This validation procedure plays a pivotal role in nonlinear dynamics and systems theory, establishing the reliability and stability of novel methods.

Keywords: *two-dimensional Volterra integral equations of the first and second kind; collocation method; Taylor polynomials; error analysis.*

Mathematics Subject Classification (2010): 45D05, 45L05, 65R20, 70K99, 93A99.

1 Introduction

The nonlinear 2D VIE of the first kind, which includes an unknown function u, can be represented in a standard form as follows:

$$\int_0^\tau \int_0^z \kappa(\tau, z, t, s) H(u(t, s)) ds dt = f(\tau, z), \quad (\tau, z) \in D, \tag{1}$$

where D is a subset of $\mathbb{R}^2$ defined as $[0, T] \times [0, Z]$, f and κ are smooth functions on their corresponding domains. Additionally, H is a continuous inverse function that is

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