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Solvability of Nonlinear Elliptic Problems with Degenerate Coercivity in Weighted Sobolev Space

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Abstract: In this paper, we investigate the existence of our entropy solution for the nonlinear elliptic equation

$$-\operatorname{div}[\omega(x)a(x,u,\nabla u)] = f - \operatorname{div} F, \quad \text{in } \Omega,$$

in the setting of the weighted Sobolev space $W_0^{1,p}(\Omega,\omega)$. We focus on the case where the operator has a degenerate coercivity and $f \in L^1(\Omega)$, $F \in [L^{p'}(\Omega,\omega^{1-p'})]^N$.

Keywords: *nonlinear elliptic equations; degenerate coercivity; entropy solutions; weighted Sobolev spaces.*

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1 Introduction

Partial differential equations have many applications in various areas of engineering, mathematics, physics, and other applied sciences (see for instance [8, 20]). In the last years, there has been an increasing interest in the study of various mathematical problems in weighted Sobolev spaces motivated by many considerations in applications (see [1, 2, 5, 10, 11] and the references therein).

Let Ω be a bounded smooth subset of $\mathbb{R}^N$ with $N \geq 2$ and $1 < p < \infty$. We are interested in proving the existence of entropy solutions to the following elliptic Dirichlet problem:

$$(\mathcal{P}) \begin{cases} -\operatorname{div}[\omega(x)a(x,u,\nabla u)] = f - \operatorname{div} F, & \text{in } \Omega, \\ u(x) = 0, & \text{on } \partial\Omega. \end{cases}$$

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