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Global Existence for the 3-D Generalized Micropolar Fluid System in Critical Fourier-Besov Spaces with Variable Exponent

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Abstract: In this work, we study the 3-D generalized Cauchy problem of the incompressible micropolar fluid system (GMFS) in the critical variable exponent Fourier-Besov space $\mathcal{F}\dot{\mathcal{B}}^{4-\frac{3}{p(\cdot)}-2\alpha}_{p(\cdot),q}$. We establish the global well-posedness result with the initial data belonging to $\mathcal{F}\dot{\mathcal{B}}^{4-\frac{3}{p(\cdot)}-2\alpha}_{p(\cdot),q}$, where $p = p(\cdot)$ is a bounded function satisfying $p \in [2, \frac{6}{5-4\alpha}]$, $\alpha \in (\frac{1}{2}, 1]$ and $q \in [1, \frac{3}{2\alpha-1}]$.

Keywords: *global existence; 3-D generalized micropolar fluid system; variable Fourier-Besov space.*

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1 Introduction and Statement of Main Result

We investigate the generalized incompressible micropolar system in the whole space $\mathbb{R}^3$,

$$\begin{cases} \partial_t u + (\chi+\nu)(-\Delta)^{\alpha_1} u + u\cdot\nabla u + \nabla\pi - 2\chi\nabla\times w = 0, & \text{in } \mathbb{R}^3\times\mathbb{R}^+, \\ \partial_t w + \mu(-\Delta)^{\alpha_2} w + u\cdot\nabla w + 4\chi w - \kappa\nabla \operatorname{div} w - 2\chi\nabla\times u = 0, & \text{in } \mathbb{R}^3\times\mathbb{R}^+, \\ \operatorname{div} u = 0, & \text{in } \mathbb{R}^3\times\mathbb{R}^+, \\ (u,w)|_{t=0} = (u_0, w_0), & \text{in } \mathbb{R}^3. \end{cases} \tag{1}$$

The unknowns are $u = u(x,t), w = w(x,t)$ and $\pi = \pi(x,t)$ representing, respectively, the linear velocity field, the micro-rotation velocity field and the pressure field of the fluid. The nonnegative constants κ, μ, ν and χ represent the viscosity coefficients, which determine fluid physical characteristics and $\alpha_1, \alpha_2 \in (\frac{1}{2}, 1]$ are two positive constants. u_0 and w_0 represent the initial velocities and we assume that $\operatorname{div} u_0 = 0$. Recall that

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