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The Regularization Method For Solving Sub-Riemannian Geodesic Problems

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Abstract: In this paper, we used the regularization method to prove some properties of the sub-Riemannian geodesics in infinite dimension for a Hilbertian manifold. More precisely, we generalize the result obtained by S.Nikitin [14], so we prove that the sub-Riemannnian distance for the Hilbert-Schmidt distribution can be approximated by the smooth sub-Riemannian geodesics.

Keywords: *regularization method; geodesics; sub-Riemannian geometry; control problem; Hamilton's equation.*

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1 Introduction

In finite-dimension context, a sub-Riemannian distance between two fixed points is defined by the infimum length of curves connecting them and whose velocity is constrained to be tangent to sub-vector space (distribution) of the tangent space T_xM of a Riemannian manifold M, where $x \in M$. Such curves are called horizontal. The distance is finite if every pair of points can be connected by at least one horizontal curve and is achieved on the curves of minimal length. Finding a length minimizer is an optimal control problem, the extremals of this problem are called the sub-Riemannian geodesics. According to the Pontryagin maximum principle [6, 10, 15, 16], the optimal curves are of two types: abnormal curves and normal geodesics which are the projections of the Hammiltonian trajectories. In [14], in finite dimension, S.Nikitin presented conditions under which the sub-Riemannian distance can be measured by an infinitely smooth sub-Riemannian

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