



# Asymmetric Duffing Oscillator: Jump Manifold and Border Set

J. Kyzioł and A. Okniński \*

*Politechnika Świętokrzyska, Al. 1000-lecia PP 7, 25-314 Kielce, Poland*

Received: October 24, 2022; Revised: December 22, 2022

**Abstract:** The jump phenomenon, present in the forced asymmetric Duffing oscillator, is studied using the known steady-state asymptotic solution. The main result consists in construction of a new mathematical object – a jump manifold – encoding global information about all possible jumps. The jump manifold is computed for the forced asymmetric Duffing oscillator, and several examples of jumps are calculated, showing the advantages of the method.

**Keywords:** *metamorphoses of amplitude-frequency curves; jump phenomenon.*

**Mathematics Subject Classification (2010):** 34C05, 34C25, 34E05, 37G35, 70K30.

## 1 Introduction

In this work, we study steady-state dynamics of the forced asymmetric Duffing oscillator governed by the equation

$$\ddot{y} + 2\zeta\dot{y} + \gamma y^3 = F_0 + F \cos(\Omega t), \quad (1)$$

which has a single equilibrium position and a corresponding one-well potential [1], where  $\zeta$ ,  $\gamma$ ,  $F_0$ ,  $F$  are parameters and  $\Omega$  is the angular frequency of the periodic force. This dynamical system in particular and Duffing-type equations in general, which can be used to describe pendulums, vibration absorbers, beams, cables, micromechanical structures, and electrical circuits, have a long history [2]. The equation of motion (1) can describe several nonlinear phenomena such as various nonlinear resonances, symmetry breaking, chaotic dynamics, period-doubling route to chaos, multistability and fractal dependence on initial conditions, and jumps [1–6].

---

\* Corresponding author: <mailto:fizao@tu.kielce.pl>