Existence of Weak Solutions for a Class of $(p(b(u)), q(b(u)))$-Laplacian Problems

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Abstract: In this paper, we consider the existence of weak solutions for some parabolic $(p(b(u)), q(b(u)))$-Laplacian problem when (p, q) is a nonlocal quantity. The novelty of this work is the study of some problems involving the (p, q)-Laplacian operator in the nonlocal case. The motivation to study these nonlocal problems relies in the fact that in reality, the measurements of some physical quantities are not made pointwise but through some local averages.

Keywords: $(p(b(u)), q(b(u)))$-*Laplacian; weak solutions; parabolic problem, generalised Sobolev spaces.*

Mathematics Subject Classification (2010): 35J60, 35J05, 35-XX, 35Kxx.

1 Introduction

The study of partial differential equations involving the (p, q)-Laplacian generalized several types of problems not only in physics, but also in biophysics, plasma physics, and in the study of chemical reactions. These problems appear, for example, in a general reaction–diffusion system

$$u_t = -\operatorname{div}\left[\left(a_p|\nabla u|^{p-2} + b_q|\nabla u|^{q-2}\right)\nabla u\right] + f(x, u),$$

where $a_p, b_q \in \mathbb{R}^+$ are some positive constants, the function u generally describes the concentration, the term $\operatorname{div}\left[\left(a_p|\nabla u|^{p-2} + b_q|\nabla u|^{q-2}\right)\nabla u\right]$ corresponds to the diffusion with coefficient $D(u) = a_p|\nabla u|^{p-2} + b_q|\nabla u|^{q-2}$, and $f(x, u)$ is the reaction term related to the source and loss processes. In general, the reaction term $f(x, u)$ has a polynomial form with respect to the concentration u.

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