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Exact Controllability of the Reaction-Diffusion Equation under Bilinear Control

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Abstract: The goal of this paper is to study the global exact controllability of a reaction-diffusion equation in a bounded domain with Dirichlet boundary conditions. We will first consider the case of bilinear equation, then we identify a set of target states that can be exactly reached at any a priori given time. This result is then applied to prove the exact controllability of semilinear reaction-diffusion equation under distributed controls. The approach is constructive and based on linear semigroup theory and null controllability properties of linear problems.

Keywords: *exact controllability; reaction-diffusion equation; bilinear control.*

Mathematics Subject Classification (2010): 35K57, 35K58, 93C20.

1 Introduction

This paper deals with the controllability of the following semilinear reaction-diffusion equation:

$$\begin{cases} y_t = \Delta y + q(x,t)y + f(y), & \text{in } Q_T \ \ (T>0), \\ y(0,t) = 0, & \text{on } \Sigma_T, \\ y(x,0) = y_0(x), & \text{in } \Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n, n \geq 1$ with a boundary $\partial\Omega$, $Q_T = \Omega \times (0,T)$ and $\Sigma_T = \partial\Omega \times (0,T)$. Here, $q \in L^\infty(Q_T)$ is a control function with the corresponding solution $y = y(x,t)$. The nonlinearity $f : \mathbb{R} \longrightarrow \mathbb{R}$ is assumed to be a Lipschitz function such that $f(0) = 0$.

In terms of applications, the equation like (1) provides the practical description of various real problems such as chemical reactions, nuclear chain reactions, biomedical

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