



# Controllability of Dynamic Equations with Memory

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**Abstract:** In this work, we consider a control system governed by a dynamic equation with memory. We obtain conditions under which the system is approximately controllable and approximately controllable on free time. In order to do this, we use a technique developed by Bashirov et al. [4–6], where we can avoid fixed point theorems. But first of all, we prove the existence and uniqueness of solutions of the system and after that, we prove the prolongation of solutions under some additional condition. Finally, we present several examples to illustrate the applicability of our results.

**Keywords:** *controllability; semilinear dynamic equations; memory; time scales.*

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## 1 Introduction

Control theory addresses how a system can be modified through feedback, in particular, how an arbitrary initial state can be directed either exactly or approximately close to a given final state using a control in a set of admissible controls. In the last decades, control theory of dynamic equations on time scales has attracted the attention of several researches, because this is a powerful tool that allows to study from a unified point of view controllability of continuous systems, discrete systems, systems in which the time variable can vary both continuously and discretely, as well as other types of time variables. Among the works made, we can cite Bartosiewicz [1] who explored linear positive control systems, Bartosiewicz and Pawłuszewicz [2, 3] reviewed linear systems, Janglajew and Pawłuszewicz [15] analyzed constrained local controllability of linear dynamic systems,

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