Nonlinear Dynamics and Systems Theory, **22** (4) (2022) 400–406

On the Dynamics of a Class of Planar Differential Systems

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Received: May 24, 2021; Revised: October 13, 2022

Abstract: In this work, we discuss the existence of the first integral and no-existence of limit cycles for a class of Kolmogorov differential systems. As an application, we give an example to illustrate our results.

Keywords: *Kolmogorov systems; periodic orbits; limit cycle; integrability.*

Mathematics Subject Classification (2010): 37C07, 34C05, 34C07, 37K10.

1 Introduction

By definition, a two-dimensional real planar Kolmogorov system will be a differential system of the form

$$\begin{cases} \dot{x} = xf_1(x,y), \\ \dot{y} = yf_2(x,y), \end{cases} \tag{1}$$

where f_1, f_2 are real functions in the two variables x and y and the dot denotes derivative with respect to the time (t) variable. There are many natural phenomena which can be modelled by the Kolmogorov systems in mathematical ecology and population dynamics, see for example [5, 10].

Kolmogorov models are widely used in ecology to describe the interaction between two populations, and a limit cycle corresponds to an equilibrium state of the system. In the qualitative theory of dynamical systems, see [2, 4, 5, 11], one of the most important problems is the study of the limit cycles of planar dynamical systems (1). The definition of limit cycles appeared in the works of Poincaré [9], the statement of the 16-th Hilbert's problem, and the discovery by Liénard [8]. A limit cycle of a planar vector field given

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