Nonlinear Dynamics and Systems Theory, **22** (4) (2022) 355–366

Functional Differential Inclusions with Unbounded Right-Hand Side in Banach Spaces

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Received: March 27, 2022; Revised: September 15, 2022

Abstract: In this work, we provide a reduction method that solves functional differential inclusion in Banach spaces, that is, when the right-hand side contains a finite delay. We consider the case when the set-valued mapping takes nonempty closed nonconvex and unnecessarily bounded values, we use the notion of $\lambda - H$ Lipschitzness assumption instead of the standard Lipschitz condition, known as a truncation. An application to a dynamical system governed by a delayed perturbed sweeping process is given, such problems are well-posed for differential complementarity systems and vector hysteresis problems.

Keywords: *nonconvex differential inclusion; reduction; delay; unboundedness; λ-Hausdorff distance.*

Mathematics Subject Classification (2010): 93C10, 34A60.

1 Introduction

Let τ, T be two non-negative real numbers, E be a separable Banach space equipped with the norm $\|\cdot\|$, $\mathcal{C}_0 := \mathcal{C}_E([-\tau, 0])$ (resp. $\mathcal{C}_T := \mathcal{C}_E([-\tau, T])$) be the Banach space of all continuous mappings from $[-\tau, 0]$ (resp. $[-\tau, T]$) to E equipped with the norm of uniform convergence. Let $\Pi : [0, T] \times \mathcal{C}_0 \rightrightarrows E$ be a set-valued mapping with nonempty closed values. In this work, we study the existence of solutions for the following differential inclusion with delay:

$$(DP) \quad \begin{cases} \dot{u}(t) \in \Pi(t, Z(t)u) & \text{a.e. } t \in [0, T], \\ u(t) = \psi(t), & t \in [-\tau, 0], \end{cases}$$

where $\psi \in \mathcal{C}_0$ and $Z(t) : \mathcal{C}_T \longrightarrow \mathcal{C}_0$ is defined by $(Z(t)u)(s) = u(t + s), \forall s \in [-\tau, 0]$. In [9], Fryszkowski proved an existence result for (DP) when Π is an integrably bounded

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