Equivalent Conditions and Persistence for Uniformly Exponential Dichotomy

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Abstract: The purpose of this paper is to provide equivalence conditions of existing conditions for the uniformly exponential dichotomy of strongly continuous quasi groups (C_0-quasi groups) on Banach spaces. There are four equivalent conditions for the existence of uniformly exponential dichotomy in the used classes of continuous and integrable function spaces over $\mathbb{R}$. Each condition emphasizes the existence and uniqueness of mild solutions of the corresponding inhomogeneous equation on the corresponding space in the C_0-quasi group term. The results are parallel with the dichotomy for the evolution family. Moreover, a small time-dependent perturbation of the infinitesimal generator of the C_0-quasi groups persists the uniformly exponential dichotomy. The results are also motivated by illustrative examples.

Keywords: *strongly continuous quasi semigroup; uniformly exponential dichotomy; mild solution; time-dependent perturbation; persistence.*

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1 Introduction

As a generalization of exponential stability and dichotomy for the evolution family [1,2], the Dichotomy Theorem of C_0-quasi groups on Banach spaces has just been developed in [3], see Theorem 4. The theorem implies that a uniformly exponential dichotomy of the C_0-quasi groups on Banach spaces X is equivalent to the spectral property of the corresponding evolution semigroup on $L_p(\mathbb{R}, X)$. Besides, the uniformly exponential dichotomy is also equivalent to the existence and uniqueness of Green's function for the quasi group, Theorem 9 of [3]. The uniformly exponential stability in this paper refers to the term in [4–6].

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