Solvability of Equations with Time-Dependent Potentials

M. Mardiyana [1*], S. Sutrima [2], R. Setiyowati [2] and R. Respatiwulan [3]

[1] *Department of Education Mathematics, University of Sebelas Maret, Ir. Sutami, no.36 A Kentingan, 57126, Surakarta, Indonesia.*
[2] *Department of Mathematics, University of Sebelas Maret, Ir. Sutami, no.36 A Kentingan, 57126, Surakarta, Indonesia.*
[3] *Department of Statistics, University of Sebelas Maret, Ir. Sutami, no.36 A Kentingan, 57126, Surakarta, Indonesia.*

Abstract: This paper is devoted to the solution of equations with time-dependent potential, at which the heat and wave equations are taken as prototype problems. The method of separating variables failed to be applied to the equations. The well-posedness of the problems is justified by strongly continuous quasi semigroups. The positive solution of the heat equations is conditioned by the maximum principle depending on the potential. For the wave equations, the bounded potentials imply the well-posedness of the problems. Further, firstly approximate solutions can be schemed. The heat and wave equations with specific potentials are considered.

Keywords: *strongly continuous quasi semigroup; heat equation; wave equation; time-dependent potential; well-posedness.*

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1 Introduction

Some phenomena of reaction-diffusion in physical systems have models as equations with time-dependent potentials [1, 2]. In general, they take the forms of nonautonomous Cauchy problems (NCP) on Banach spaces [3–6],

$$
\begin{aligned}
\dot{u}(t) &= A(t)u(t), \qquad t \geq 0, \\
u(0) &= u_0, \qquad\quad u_0 \in X,
\end{aligned}
\tag{1}
$$

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* Corresponding author: `mardiyana@staff.uns.ac.id`