Asymptotic Analysis of a Nonlinear Elliptic Equation with a Gradient Term

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Abstract: The main purpose of the present paper is to study the equation

$$div(|\nabla u|^{p-2}\nabla u) + \alpha u + \beta x.\nabla u + |u|^{q-1}u = 0, \quad x \in \mathbb{R}^N,$$

where $p > 2$, $q > 1$, $N \geqslant 1$, $\alpha > 0$ and $\beta > 0$. We investigate the structure of radial solutions and we present the asymptotic behavior of positive solutions near infinity. The study depends strongly on the sign of $N\beta - \alpha$ and the comparison between the three determining values $\dfrac{\alpha}{\beta}$, $\dfrac{p}{q+1-p}$ and $\dfrac{N-p}{p-1}$. More precisely, we prove under some assumptions that there exists a positive solution u which has the following behavior near infinity:

$$u(r) \underset{+\infty}{\sim} \left(N - p - \frac{\alpha}{\beta}(p-1)\right)^{\frac{1}{q+1-p}} \left(\frac{\alpha}{\beta}\right)^{\frac{p-1}{q+1-p}} r^{-\alpha/\beta}.$$

Keywords: *nonlinear elliptic equation; radial self-similar solution; global existence; energy function; asymptotic behavior; equilibrium point; nonlinear dynamical systems.*

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