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Lyapunov-Type Inequalities for a Fractional Boundary Value Problem with a Fractional Boundary Condition

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Abstract: In this paper, we consider a linear fractional differential equation with fractional boundary conditions. First, by obtaining Green's function, we derive the Lyapunov-type inequalities for such boundary value problems. Furthermore, we use the contraction mapping theorem to study the existence of a unique solution to a nonlinear problem.

Keywords: *fractional boundary value problem, Lyapunov-type inequalities, Green's function, contraction mapping theorem, uniqueness and existence of solutions.*

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1 Introduction

For the second-order linear differential equation

$$u'' + q(t)u = 0, \quad t \in (a, b) \tag{1}$$

with $q \in C([a, b], \mathbb{R})$, it is known that if (1) has a nontrivial solution u with $u(a) = u(b) = 0$, then

$$\int_a^b |q(t)|\, dt > \frac{4}{b-a}. \tag{2}$$

This result is known as the Lyapunov inequality, see [1, 22].

It was first noticed by Wintner [28] and later by several other authors that inequality (2) can be improved by replacing $|q(t)|$ by $q_+(t) := \max\{q(t), 0\}$, the nonnegative part of $q(t)$.

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