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The Geometry of Mass Distributions

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Abstract: Geometrical characteristics of mass distributions are defined and the relation with classical mechanics and general relativity is described. The classical stability of closed geodesic trajectories on surfaces of arbitrary genus is established. An iterative procedure for solving the N-body problem to a high degree of precision is introduced through a complexity minimization method.

Keywords: *center; geodesics; geometrical complexity; N-body problem.*

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1 Introduction

The equations of classical mechanics and general relativity describe the motion of particle in a geometry of three or four dimensions. The potential in general relativity is derived from the curvature of space-time which results from the energy-momentum tensor or mass distribution. The effect of the geometry of the mass distribution on the dynamics will be considered. A geometrical median will be given and verified for various curves and surfaces. It is proven in the two theorems of Section 2 that the geometrical median of a curve is located on the curve if it is a straight line in Euclidean space and a geodesic in curved space. These theorems remain valid for the barycentre which coincides with the center of mass of a uniform distribution. The role of the center of the mass distribution then will be described in classical mechanics and general relativity. It is known that mass distributions tend towards the center [8]. The local stability of geometrical configurations under the gravitational potential will follow for geodesics.

The stability of geodesics that can be identified with strings on a surface is considered. Given the tendency of uniform mass distributions towards the center of a geometrical configuration, it follows from the theorems of Section 2 that only closed geodesics will

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