On the Equivalence of Lorenz System and Li System

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Abstract: The question of the equivalence of various Lorenz-like systems has been recently discussed, it has been found that with the help of various transformations it is possible to reduce such systems to the same form. In this paper, we show that the Lorenz system and the Li system are topologically equivalent. However, in a recent work it was shown that there is a homothetic transformation which converts the Li system into the Lorenz system and, therefore, all the dynamical behavior exhibited by the Li system is also present in the Lorenz system. Consequently, the results obtained in the papers devoted to the study of the Li system unnecessarily duplicate the scientific literature, while it can be trivially derived from the corresponding results on the Lorenz system.

Keywords: *Lorenz system; Li system; homothetic transformation; topological equivalence.*

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1 Introduction

In 1963, E.N. Lorenz [9] discovered chaos in a simple system of three autonomous ordinary differential equations

$$\begin{cases} X' = \sigma(Y - X), \\ Y' = \rho X - Y - XZ, \\ Z' = -\beta Z + XY, \end{cases} \tag{1}$$

where σ, ρ and β are real parameters, the system is chaotic on a small subset $\{\sigma, \rho, \beta\} = \{10, 28, \frac{8}{3}\}$. The Lorenz system is the first mathematical and physical model of chaos. Since the introduction of the Lorenz system, which attracted much attention from research teams, many other chaotic systems (generally called Lorenz-like systems) have

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