Comprehensive Description of Solutions to Semilinear Sectorial Equations: an Overview

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Abstract: Description of all possible types of behavior, or evolution, of solutions to a semilinear sectorial equation is given. The phase space is divided into separate regions containing bounded solutions, grow-up solutions and those which blow up in a finite time. An overview of results concerning the typical situation when solutions of various types of behavior coexist is given and illustrated by chosen examples of reaction-diffusion equations.

1 Introduction

This paper is devoted to the fundamental question connected with solutions of semilinear sectorial equations (1) being generalizations of parabolic equations: *Provided that a local in time solution exists, what is the expected future for the rest of its existence?*

It is known from the classical references, such as [20, Chapter I], that, in general, there are three potential forms of the further evolution of such solutions:
– the local solution may *blow up*, which means that its phase space norm becomes unbounded in a finite time; in general, it can be a consequence of unboundedness of the values of the solution or the values of some of its derivatives, even though the solution itself may stay bounded in the L^∞−norm,
– the local solution may *grow up*, that is, it will exist for all positive times, while some of its norms will become unbounded as $t \to \infty$,

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