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Capacity and Anisotropic Sobolev Spaces with Zero Boundary Values

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Abstract: The aim of this work is to study the capacity theory in anisotropic Sobolev spaces. In particular, we will give main properties of capacity, including monotonicity, countable subadditivity and several convergence results. Moreover, we will define the anisotropic Sobolev space with zero boundary values $B_0^{1,\vec{p}}(\Omega)$, where Ω is an open bounded set of $\mathbb{R}^N (N \geq 2)$, $\vec{p} = (p_0, p_1, ..., p_N)$ and $1 < p_0, p_1, ..., p_N < \infty$. This allows us to prove that the Dirichlet energy integral has a minimizer in the anisotropic Sobolev space with zero boundary values $B_0^{1,\vec{p}}(\Omega)$.

Keywords: *capacity; anisotropic Sobolev space with zero boundary values; Dirichlet energy.*

Mathematics Subject Classification (2010): 31B15, 31C15, 46E35, 70Kxx, 93XX.

1 Introduction

The notion of capacity is an essential tool in the study of nonlinear potential theory, which allows us to measure sets more precisely than the usual Lebesgue measure, to see that functions are better defined almost everywhere (quasi everywhere). Capacities play a key role in the study of solutions of partial differential equations, for example, Boccardo et al. studied in [6] the existence and non existence of solutions of the following problem:

$$(\mathcal{P}) \begin{cases} -\triangle u + u \mid \nabla u \mid^2 = \mu & in \ \ \Omega, \\ u = 0 & on \ \ \partial\Omega, \end{cases}$$

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