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Existence of Solutions for the Debye-Hückel System with Low Regularity Initial Data in Critical Fourier-Besov-Morrey Spaces

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Abstract: This paper is devoted to studying the existence of solutions for the Cauchy problem of the Debye-Hückel system with low regularity initial data in critical Fourier-Besov-Morrey spaces. We show that there exists a unique local solution if the initial data belong to the Fourier-Morrey-Besov space $\mathcal{FN}^{-2+\frac{n}{p'}+\frac{\lambda}{p}}_{p,\lambda,q} \times \mathcal{FN}^{-2+\frac{n}{p'}+\frac{\lambda}{p}}_{p,\lambda,q}$, and furthermore, if the initial data are sufficiently small, then the solution is global.

Keywords: *Debye-Hückel system; local existence; global existence; Littlewood-Paley theory; Fourier-Morrey-Besov spaces.*

Mathematics Subject Classification (2010): 35K45, 35Q99, 70k99, 93-00.

1 Introduction

In this paper, we consider the following Cauchy problem for the Debye-Hückel system in $\mathbb{R}^n \times \mathbb{R}^+$:

$$\begin{cases} \partial_t v = \Delta v - \nabla \cdot (v \nabla \phi) & \text{in } \mathbb{R}^n \times (0,\infty), \\ \partial_t w = \Delta w + \nabla \cdot (w \nabla \phi) & \text{in } \mathbb{R}^n \times (0,\infty), \\ \Delta \phi = v - w & \text{in } \mathbb{R}^n \times (0,\infty), \\ v(x,0) = v_0(x), \quad w(x,0) = w_0(x) & \text{in } \mathbb{R}^n, \end{cases} \tag{1}$$

where the unknown functions $v = v(x,t)$ and $w = w(x,t)$ denote densities of the electron and the hole in electrolytes, respectively, $\phi = \phi(x,t)$ denotes the electric potential, $v_0(x)$ and $w_0(x)$ are the initial data. Throughout this paper, we assume that $n \geq 2$.

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