Nonlinear Dynamics and Systems Theory, **21** (4) (2021) 337–359

Local Analysis for a Mutual Inhibition in Presence of Two Viruses in a Chemostat

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Received: March 25, 2021; Revised: July 25, 2021

Abstract: A competition with mutual inhibition is a form of direct competition between the populations of two species where each actively inhibits the other. In this paper, we consider a mathematical system of ordinary differential equations describing two species, with mutual inhibition, competing for a limiting substrate in the presence of two viruses. A detailed local qualitative analysis of the restriction of the system to the attractor set is carried out. We prove that for general nonlinear response functions, the Competitive Exclusion Principle is still fulfilled so that at most one species can survive. Initial species concentrations are important in determining which is the winning species. The results obtained were validated by numerical simulations using Matlab software.

Keywords: *chemostat; competition; reversible inhibition; virus; local analysis; competitive exclusion principle.*

Mathematics Subject Classification (2010): 34D20, 37C75, 65L07, 65L20, 92B05, 92B10, 93B18, 93D20.

1 Introduction

A chemostat is a laboratory device (bioreactor) in which organisms grow on the available nutrient in a controlled manner. In many applications, it is simply a vessel used as a wastewater treatment process [18]. In ecology, it refers to an artificial lake for the continuous culture of bacteria which allows us to analyse inter-specific interactions between bacteria. A large number of mathematical studies have been published [18]. The most used mathematical system modelling the bacterial competition for a single obligate limiting substrate predicts competitive exclusion [12], that is, at least one competitor bacteria loses the competition [18]. Hsu et al. [15] in 1977, were among the first to study the problem of competition in a chemostat. They considered n populations in competition for the same nutrient and showed that competitive exclusion was verified, namely, the competitor which is better at using the substrate in small quantities survives and the others are extinguished. In the case of nonmonotonic growth functions, Butler and Wolkowicz [2] in 1985, also verified the competitive exclusion principle. In 1992, Wolkowicz and Lu [19] used Lyapunov functions to also verify the competitive exclusion principle in the case of general shape-growth functions, but with different mortality rates. For each species, the competitive exclusion principle was further checked (the resulting equilibrium being globally stable). Li [16] recently extended this result to

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