Nonlinear Dynamics and Systems Theory, **21** (3) (2021) 326–336

# Mathematical Study of a Modified SEIR Model for the Novel SARS-Cov-2 Coronavirus

Abdelhamid Zaghdani *

*Northern Border University, Faculty of Arts and Science, Rafha, P.O 840, Saudi Arabia. University of Tunis, Boulevard du 9 avril, 1939, Tunis. Department of Mathematics, Ensit, Taha Hussein Avenue, Montfleury, Tunis, Tunisia.*

Received: January 22, 2021; Revised: May 15, 2021

**Abstract:** In this paper, a modified SEIR mathematical model for the coronavirus infected disease-2019 (COVID-19) has studied. We named this model the SEIQRC model and analyzed the stability mathematically. A qualitative analysis of the local and global stability of equilibrium points is carried out. It is shown that the disease-free equilibrium is globally asymptotically stable when the basic reproduction number $\mathcal{R}_0 \leq 1$ and the disease-persistence equilibrium is globally asymptotically stable when $\mathcal{R}_0 > 1$.

**Keywords:** *COVID-19; coronavirus; SEIQRC model; local and global stability; direct Lyapunov method; Lasalle's invariance principle.*

**Mathematics Subject Classification (2010):** 34D23, 35N25, 37B25, 49K40, 60H10, 65C30, 91B70.

## 1 Introduction

The novel Coronavirus was detected in China and a few months later it spreaded in the countries all over the world. Covid-19 contamination can be transmitted to a person from a contaminated person, a contaminated dry surface, through the nose or mouth. In March 2020, the World Health Organization declared the Covid-19 a global pandemic. For today, the novel Coronavirus caused tens of thousands of deaths and a few million cases of infections. It can be classified as the third highly pathogenic human Coronavirus appearing in the past two decades. Since its appearance, several scientific researchers have been interested in studies of various problems related to this novel Coronavirus [2, 7, 13].

* Corresponding author: mailto:hamido20042002@yahoo.fr

In this paper, we study the following modified mathematical model of ODEs proposed first in [5]:

$$\begin{cases} \dot{S} = \mu N_T - \beta S I N_T - (\alpha + \mu)S, \\ \dot{E} = \beta S I N_T - (\gamma + \mu)E, \\ \dot{I} = \gamma E - (\delta + \mu)I, \\ \dot{Q} = \delta I - (\lambda + \mu)Q, \\ \dot{R} = \lambda Q - \mu R, \\ \dot{C} = \alpha S - \mu C \end{cases} \tag{1}$$

with the positive initial condition $(S(0), E(0), I(0), Q(0), R(0), C(0)) \in \mathbb{R}^6_+$ , where $S$ is the susceptible population, $E$ is the exposed population, $I$ is the infected population, $Q$ is the population under quarantine (reported infected cases), $R$ is the recovered population and $C$ is the confined susceptible population.

The outline of this paper is as follows. In Section 2, some properties of the system (1) are given. Section 3 is devoted to the calculation of the basic reproduction number $\mathcal{R}_0$ using the next generation matrix method to assess the transmissibility of the novel Coronavirus Covid-19. The analysis of the local and global stability of equilibrium points is presented in Sections 4 and 5, respectively. It is shown that the disease-persistence (endemic) equilibrium is globally asymptotically stable when $\mathcal{R}_0 > 1$. However, when $\mathcal{R}_0 \leq 1$, then the disease-free equilibrium is globally asymptotically stable. Finally, Section 6 is done to present some numerical tests confirming the obtained theoretical results.

## 2 Properties of the Mathematical Model

The parameters of the model (1) are the protection rate $\alpha$, the infection rate $\beta$, the incubation rate $\gamma$, the quarantine rate $\delta$, the natural mortality rate $\mu$ (which is proportional to the birth rate) and the recovery rate $\lambda$. Define $\bar{P} = (\frac{\mu N_T}{\alpha+\mu}, 0, 0, 0, 0, \frac{\alpha N_T}{\alpha+\mu})$ as the disease free equilibrium point.

**Proposition 2.1**

1. *For every given initial condition $(S(0), E(0), I(0), Q(0), R(0), C(0))$ in $\mathbb{R}^6_+$ , system (1) admits a bounded solution with positive components defined for all $t > 0$.*
2. *The set $\Omega_1 = \{(S, E, I, Q, R, C) \in \mathbb{R}^6_+ \ / \ S+E+I+Q+R+C = N_T\}$ is a positively invariant attractor for system (1).*

***Proof.***

1. The solution is positive due to the fact that since $S = 0$, one has $\dot{S} = \mu N_T > 0$; if $E = 0$, then $\dot{E} = \beta S I N_T > 0$; once $I = 0$, then $\dot{I} = \gamma E > 0$; if $Q = 0$, then $\dot{Q} = \delta I > 0$; if $R = 0$, then $\dot{R} = \lambda Q > 0$; and if $C = 0$, then $\dot{C} = \alpha S > 0$.

   The boundedness of solutions of system (1) can be proved by adding all equations of system (1), and then one obtains, for $T = S + E + I + Q + R + C - N_T$, the following equation for the totality of individuals:

$$\dot{T} = \dot{S} + \dot{E} + \dot{I} + \dot{Q} + \dot{R} + \dot{C} = \mu N_T - \mu S - \mu E - \mu I - \mu Q - \mu R - \mu C = -\mu T.$$

Then

$$\begin{array}{rcl} S(t) & + & E(t)+I(t)+Q(t)+R(t)+C(t) \\ & = & N_T+(S(0)+E(0)+I(0)+Q(0)+R(0)+C(0)-N_T)e^{-\mu t}. \end{array} \tag{2}$$

Then the boundedness of the solution of system (1) holds since all compartments of $T$ are positive.

2. One can easily deduce from (2) that the set $\Omega_1$ is a positively invariant attractor for system (1).

## 3 Computation of the Basic Reproduction Number by the Next Generation Matrix Method

For determining the reproduction number of (1), we use the next generation matrix method proposed by Diekmann, et al. [3] and elaborated by van den Driessche and Watmough [6] for an ODE compartmental model.

In (1), the disease free-equilibrium is $\bar{P}=(\frac{\mu N_T}{\alpha+\mu},0,0,0,0,\frac{\alpha N_T}{\alpha+\mu})$ and the compartments containing infected individuals are $X=(X_1,X_2,X_3)=(E,I,Q)$. Using the generation matrix method [3,6], consider these equations written in the form $\dot{X}_i=\mathcal{F}_i(X)-\mathcal{V}_i(X)$ for $i=1,2,3$.

Now define $F=[\frac{\partial \mathcal{F}_i(\bar{P})}{X_j}]$ and $V=[\frac{\partial \mathcal{V}_i(\bar{P})}{X_j}]$ for $1\leq i,j\leq 3$. Thus the reproduction number $\mathcal{R}_0$ is the spectral radius of the matrix $FV^{-1}$ and we have

$$\mathcal{R}_0=\rho(FV^{-1}).$$

As in [6], we have the following theorem.

**Theorem 3.1** *If $\bar{P}$ is a DFE of the system $\dot{X}_i=\mathcal{F}_i(X)-\mathcal{V}_i(X)$, then $\bar{P}$ is locally asymptotically stable if $\mathcal{R}_0=\rho(FV^{-1})<1$, but unstable if $\mathcal{R}_0>1$.*

Now, we have the following theorem for the reproduction number $\mathcal{R}_0$.

**Theorem 3.2** *The reproduction number of (1) is given by*

$$\mathcal{R}_0=N_T\sqrt{\frac{\gamma\beta\mu}{(\alpha+\mu)(\gamma+\mu)(\delta+\mu)}}. \tag{3}$$

***Proof.*** According to the next generation matrix method, we have

$$F=\begin{pmatrix} 0 & \frac{\beta\mu}{\alpha+\mu}N_T^2 & 0 \\ \gamma & 0 & 0 \\ 0 & \delta & 0 \end{pmatrix} \quad \text{and} \quad V=\begin{pmatrix} \gamma+\mu & 0 & 0 \\ 0 & \delta+\mu & 0 \\ 0 & 0 & \lambda+\mu \end{pmatrix}.$$

Then

$$FV^{-1}=\begin{pmatrix} 0 & \frac{\beta\mu N_T^2}{(\alpha+\mu)(\delta+\mu)} & 0 \\ \frac{\gamma}{\alpha+\mu} & 0 & 0 \\ 0 & \frac{\delta}{\delta+\mu} & 0 \end{pmatrix}.$$

The basic reproduction number for model (1) is given by the spectral radius of the matrix $FV^{-1}$ and so

$$\mathcal{R}_0 = N_T \sqrt{\frac{\gamma\beta\mu}{(\alpha+\mu)(\gamma+\mu)(\delta+\mu)}}.$$

## 4 Local Stability

**Theorem 4.1 (1)** *If $\mathcal{R}_0 < 1$, then the disease free equilibrium $\bar{P}$ is locally asymptotically stable.*

**(2)** *If $\mathcal{R}_0 \geq 1$, then the disease free equilibrium $\bar{P}$ is unstable.*

***Proof.*** The Jacobian matrix $J$ evaluated at $\bar{P} = (\frac{\mu N_T}{\alpha+\mu}, 0, 0, 0, 0, \frac{\alpha N_T}{\alpha+\mu})$ is given by

$$\bar{J} = \begin{pmatrix} -\alpha-\mu & 0 & -\frac{\beta}{\alpha+\mu}\mu N_T^2 & 0 & 0 & 0 \\ 0 & -\gamma-\mu & \frac{\beta}{\alpha+\mu}\mu N_T^2 & 0 & 0 & 0 \\ 0 & \gamma & -\delta-\mu & 0 & 0 & 0 \\ 0 & 0 & \delta & -\lambda-\mu & 0 & 0 \\ 0 & 0 & 0 & \lambda & -\mu & 0 \\ \alpha & 0 & 0 & 0 & 0 & -\mu \end{pmatrix}.$$

The characteristic equation is

$$\begin{aligned}
\bar{P}(X) &= (X+\alpha+\mu)(X+\lambda+\mu)(X+\mu)^2 \begin{vmatrix} -\gamma-\mu-X & \frac{\beta}{\alpha+\mu}\mu N_T^2 \\ \gamma & -\delta-\mu-X \end{vmatrix} \\
&= (X+\alpha+\mu)(X+\lambda+\mu)(X+\mu)^2 \left( (\gamma+\mu+X)(\delta+\mu+X) - \frac{\gamma\beta}{\alpha+\mu}\mu N_T^2 \right) \\
&= (X+\alpha+\mu)(X+\lambda+\mu)(X+\mu)^2 \\
&\quad \times \left( X^2 + (\gamma+2\mu+\delta)X + (\gamma+\mu)(\delta+\mu) - \frac{\gamma\beta}{\alpha+\mu}\mu N_T^2 \right) \\
&= (X+\alpha+\mu)(X+\lambda+\mu)(X+\mu)^2 \\
&\quad \times \left( X^2 + (\gamma+2\mu+\delta)X + (\gamma+\mu)(\delta+\mu)(1-\mathcal{R}_0)(1+\mathcal{R}_0) \right) \\
&= (X+\alpha+\mu)(X+\lambda+\mu)(X+\mu)^2 \\
&\quad \times \left( X^2 + a_1 X + a_0 \right),
\end{aligned}$$

where $a_1 = (\gamma+2\mu+\delta) > 0$ and $a_0 = (\gamma+\mu)(\delta+\mu)(1-\mathcal{R}_0)(1+\mathcal{R}_0) > 0$ if $\mathcal{R}_0 < 1$. By the Routh-Hurwitz criterion, we deduce that all eigenvalues have negative real parts and then $\bar{P}$ is locally asymptotically stable if $\mathcal{R}_0 < 1$. If $\mathcal{R}_0 \geq 1$, then $a_0 \leq 0$ and there exists at least one non negative eigenvalue $\lambda$ of $J^\star$, therefore $\bar{P}$ is unstable.

**Theorem 4.2** *If $\mathcal{R}_0 > 1$, then the disease-persistence equilibrium $P^*$ is locally asymptotically stable.*

***Proof.*** Denote $P^* = (S^*, E^*, I^*, Q^*, R^*, C^*)$, then we have

$$\begin{aligned}
E^* &= \frac{\mu N_T - (\alpha+\mu)S^*}{\gamma+\mu} = \frac{\mu N_T}{\gamma+\mu} - \frac{(\alpha+\mu)}{\gamma+\mu}S^*,\\
I^* &= \frac{\gamma}{\delta+\mu}E^* = \frac{\gamma\mu N_T}{(\delta+\mu)(\gamma+\mu)} - \frac{\gamma(\alpha+\mu)}{(\delta+\mu)(\gamma+\mu)}S^*,\\
Q^* &= \frac{\delta}{\lambda+\mu}I^* = \frac{\delta\gamma\mu N_T}{(\lambda+\mu)(\delta+\mu)(\gamma+\mu)} - \frac{\delta\gamma(\alpha+\mu)}{(\lambda+\mu)(\delta+\mu)(\gamma+\mu)}S^*,\\
R^* &= \frac{\lambda}{\mu}Q^* = \frac{\lambda\delta\gamma\mu N_T}{\mu(\lambda+\mu)(\delta+\mu)(\gamma+\mu)} - \frac{\lambda\delta\gamma(\alpha+\mu)}{\mu(\lambda+\mu)(\delta+\mu)(\gamma+\mu)}S^*,\\
C^* &= \frac{\alpha}{\mu}S^*.
\end{aligned}$$

From the third equation in (1), we obtain

$$E^* = \frac{\delta+\mu}{\gamma}I^*.$$

Replacing this in the second equation of (1), we get

$$\begin{aligned}
-(\gamma+\mu)\frac{\delta+\mu}{\gamma}I^* + \beta S^* I^* N_T = 0 \quad &\Longrightarrow \quad (-(\gamma+\mu)\frac{\delta+\mu}{\gamma} + \beta S^* N_T)I^* = 0\\
&\Longrightarrow \quad -(\gamma+\mu)\frac{\delta+\mu}{\gamma} + \beta S^* N_T = 0\\
&\Longrightarrow \quad S^* = \frac{(\gamma+\mu)(\delta+\mu)}{\gamma\beta N_T}.
\end{aligned}$$

From the system given previously, and the value of $\mathcal{R}_0$, we can write

$$\begin{aligned}
I^* &= \frac{\gamma\mu N_T}{(\delta+\mu)(\gamma+\mu)} - \frac{\gamma(\alpha+\mu)}{(\delta+\mu)(\gamma+\mu)}S^*\\
&= \frac{\gamma\mu N_T}{(\delta+\mu)(\gamma+\mu)} - \frac{\alpha+\mu}{\beta N_T}\\
&= \frac{\alpha+\mu}{\beta N_T}(\mathcal{R}_0^2 - 1).
\end{aligned}$$

Now, we compute the characteristic equation of the Jacobian matrix evaluated at $P^*$.

We have

$$\begin{aligned}
P^*(X) &= -(X+\mu)^2(X+\lambda+\mu)\begin{vmatrix} -\alpha-\mu-\beta I^*N_T-X & 0 & -\beta S^*N_T \\ \beta I^*N_T & -\gamma-\mu-X & \beta S^*N_T \\ 0 & \gamma & -\delta-\mu-X \end{vmatrix} \\
&= -(X+\mu)^2(X+\lambda+\mu)\begin{vmatrix} -\alpha-\mu-\beta I^*N_T-X & 0 & -\beta S^*N_T \\ -\alpha-\mu-X & -\gamma-\mu-X & 0 \\ 0 & \gamma & -\delta-\mu-X \end{vmatrix} \\
&= -(X+\mu)^2(X+\lambda+\mu) \\
&\quad \times\big((-\alpha-\mu-\beta I^*N_T-X)(\gamma+\mu+X)(\delta+\mu+X)+\beta S^*N_T(\alpha+\mu+X)\gamma\big) \\
&= -(X+\mu)^2(X+\lambda+\mu) \\
&\quad \times\big(\big[-\alpha-\mu-\beta I^*N_T-X\big]\big[X^2+(\delta+\gamma+2\mu)X+(\delta+\mu)(\gamma+\mu)\big] \\
&\quad +\beta S^*N_T(\alpha+\mu+X)\gamma\big) \\
&= (X+\mu)^2(X+\lambda+\mu)\big(X^3+a_1X^2+a_2X+a_3\big)
\end{aligned}$$

with

$$\begin{aligned}
a_1 &= \alpha+\mu+\beta I^*N_T+\delta+\gamma+2\mu \\
&= \alpha+\mu+(\alpha+\mu)(\mathcal{R}_0^2-1)+\delta+\gamma+2\mu=(\alpha+\mu)\mathcal{R}_0^2+\delta+\gamma+2\mu, \\
a_2 &= (\alpha+\mu+\beta I^*N_T)(\delta+\gamma+2\mu)+(\delta+\mu)(\gamma+\mu)-\beta\gamma S^*N_T, \\
a_3 &= (\alpha+\mu+\beta I^*N_T)(\delta+\mu)(\gamma+\mu)-\beta\gamma(\alpha+\mu)S^*N_T.
\end{aligned}$$

Then

$$\begin{aligned}
a_2 &= (\alpha+\mu+(\alpha+\mu)(\mathcal{R}_0^2-1))(\delta+\gamma+2\mu)+(\delta+\mu)(\gamma+\mu)-(\gamma+\mu)(\delta+\mu) \\
&= [\alpha+\mu+(\alpha+\mu)(\mathcal{R}_0^2-1)][\delta+\gamma+2\mu] \\
&= (\alpha+\mu)[\delta+\gamma+2\mu]\mathcal{R}_0^2>0, \\
a_3 &= (\alpha+\mu+(\alpha+\mu)(\mathcal{R}_0^2-1))(\delta+\mu)(\gamma+\mu)-(\alpha+\mu)(\gamma+\mu)(\delta+\mu) \\
&= (\alpha+\mu)(\delta+\mu)(\gamma+\mu)\mathcal{R}_0^2-(\alpha+\mu)(\gamma+\mu)(\delta+\mu) \\
&= (\alpha+\mu)(\delta+\mu)(\gamma+\mu)(\mathcal{R}_0^2-1).
\end{aligned}$$

Thus $a_3>0$ if $\mathcal{R}_0>1$.

Now, we demonstrate that $a_1a_2>a_3$ if $\mathcal{R}_0>1$.

$$\begin{aligned}
a_1a_2-a_3 &= \Big[(\alpha+\mu)\mathcal{R}_0^2+\delta+\gamma+2\mu\Big]\Big[(\alpha+\mu)[\delta+\gamma+2\mu]\mathcal{R}_0^2\Big] \\
&\quad -(\alpha+\mu)(\delta+\mu)(\gamma+\mu)(\mathcal{R}_0^2-1) \\
&= (\alpha+\mu)\Big[\big[(\alpha+\mu)\mathcal{R}_0^2+\delta+\gamma+2\mu\big](\delta+\gamma+2\mu)\mathcal{R}_0^2\Big]
\end{aligned}$$

$$\begin{aligned}
&\quad -(\alpha+\mu)(\delta+\mu)(\gamma+\mu)(\mathcal{R}_0^2-1) \\
&\geq (\alpha+\mu)\Big[\Big(\delta+\gamma+2\mu\Big)^2\mathcal{R}_0^2-(\delta+\mu)(\gamma+\mu)\mathcal{R}_0^2+(\delta+\mu)(\gamma+\mu)\Big] \\
&\geq (\alpha+\mu)\Big[\Big((\delta+\gamma+2\mu)^2-(\delta+\mu)(\gamma+\mu)\Big)\mathcal{R}_0^2+(\delta+\mu)(\gamma+\mu)\Big].
\end{aligned}$$

Since $(\delta+\gamma+2\mu)^2-(\delta+\mu)(\gamma+\mu)>0$ for every positive parameters $\delta$, $\gamma$ and $\mu$, we conclude that $a_1a_2-a_3>0$ and the Routh-Hurwitz criterion permits to conclude.

## 5 Global Stability

**Lemma 5.1** $\Omega_2=\{(S,E,I,Q,R,C)\in\Omega_1;\ S\leq\dfrac{\mu N_T}{\alpha+\mu}\}$ *is a positively invariant attractor for system (1).*

***Proof.*** It is proved in Proposition 2.1 that the bounded set $\Omega_1$ is a positive invariant attractor set of all solutions of system (1). Now, since $\dot{S}(t)<0$ for $S(t)>\dfrac{\mu N_T}{\alpha+\mu}$, one has $\liminf S(t)\leq\dfrac{\mu N_T}{\alpha+\mu}$. This completes the proof.

**Theorem 5.1** *If $\mathcal{R}_0\leq 1$, then the disease-free equilibrium $\bar{P}$ is globally asymptotically stable.*

***Proof.*** Consider the following Lyapunov function: $Ł_1=\gamma E+(\gamma+\mu)I$. Therefore,

$$\begin{aligned}
\dot{Ł}_1 &= \gamma\dot{E}+(\gamma+\mu)\dot{I}\\
&= \gamma\Big(\beta SIN_T-(\gamma+\mu)E\Big)+(\gamma+\mu)\Big(\gamma E-(\delta+\mu)I\Big)\\
&= \gamma\beta SIN_T-(\gamma+\mu)(\delta+\mu)I\\
&\leq \frac{\gamma\beta\mu}{\alpha+\mu}IN_T^2-(\gamma+\mu)(\delta+\mu)I \text{ since } S\leq\frac{\mu N_T}{\alpha+\mu}\\
&= \Big(\frac{\gamma\beta\mu}{\alpha+\mu}N_T^2-(\gamma+\mu)(\delta+\mu)\Big)I\\
&= (\gamma+\mu)(\delta+\mu)\Big(\frac{\gamma\beta\mu}{(\alpha+\mu)(\gamma+\mu)(\delta+\mu)}N_T^2-1\Big)I\\
&= (\gamma+\mu)(\delta+\mu)\Big(\mathcal{R}_0^2-1\Big)I,\ \forall(S,E,I,Q,R,C)\in\Omega_2.
\end{aligned}$$

It follows that $\dot{Ł}_1\leq 0$ if $\mathcal{R}_0\leq 1$ with $\dot{Ł}_1=0$ only if $I=0$. Therefore, $Ł_1$ is a Lyapunov function on $\Omega_2$. Moreover, Lemma 5.1 implies that $\Omega_2$ is a compact, absorbing subset of $\mathbb{R}^6_+$, and the largest compact invariant set in $\{(S,E,I,Q,R,C)\in\Omega_2:\dot{Ł}_1=0\}$ is $\{\bar{P}\}$. Therefore, by the Lasalle invariance principle (see, for example, [11, Theorem 3.1] and [1,4,8–10,14,15] for other applications), we deduce that every solution of system (1) with the initial conditions in $\mathbb{R}^6_+$ converges to $\bar{P}$ as $t\to+\infty$.

Now, we give a result of global stability for the disease-persistence equilibrium $P^*$.

**Theorem 5.2** *The disease-persistence equilibrium $P^*$ is globally asymptotically stable if $\mathcal{R}_0>1$.*

***Proof.*** Consider the Lyapunov function

$$Ł_2 = \Big(S-S^*\ln(\frac{S}{S^*})\Big)+\Big(E-E^*\ln(\frac{E}{E^*})\Big)+\frac{(\gamma+\mu)}{\gamma}\Big(I-I^*\ln(\frac{I}{I^*})\Big).$$

$P^*$ is the global minimum of $Ł_2$. Indeed, $P^*$ is the unique internal stationary point of system (1) and the function $Ł_2$ has its minimum value $Ł_{2min}=S^*+E^*+\dfrac{(\gamma+\mu)}{\gamma}I^*$

when $S = S^*,\ E = E^*,\ I = I^*,\ Q = Q^*,\ R = R^*,\ C = C^*$, and $Ł_2(t) \to +\infty$ at the boundary of the positive quadrant.

Now, we compute the derivative of $Ł_2(t)$ along the solutions of system (1):

$$\begin{aligned}\dot{Ł}_2 &= \Big(1-\frac{S^*}{S}\Big)\dot{S} + \Big(1-\frac{E^*}{E}\Big)\dot{E} + \frac{(\gamma+\mu)}{\gamma}\Big(1-\frac{I^*}{I}\Big)\dot{I}\\ &= \Big(1-\frac{S^*}{S}\Big)\Big(\mu N_T - \beta S I N_T - (\alpha+\mu)S\Big) + \Big(1-\frac{E^*}{E}\Big)\Big(\beta S I N_T - (\gamma+\mu)E\Big)\\ &\quad + \frac{(\gamma+\mu)}{\gamma}\Big(1-\frac{I^*}{I}\Big)\Big(\gamma E - (\delta+\mu)I\Big).\end{aligned}$$

Since $\mu N_T = \beta S^* I^* N_T + (\alpha+\mu)S^*$ and $(\gamma+\mu)E^* = \dfrac{(\delta+\mu)(\gamma+\mu)}{\gamma} I^* = \beta S^* I^* N_T$, we can write

$$\begin{aligned}\dot{Ł}_2 &= \Big(1-\frac{S^*}{S}\Big)\Big(\beta S^* I^* N_T + (\alpha+\mu)S^* - \beta S I N_T - (\alpha+\mu)S\Big)\\ &\quad + \beta S I N_T - (\gamma+\mu)E - \beta S I N_T \frac{E^*}{E} + (\gamma+\mu)E^*\\ &\quad + (\gamma+\mu)E - \frac{(\delta+\mu)(\gamma+\mu)}{\gamma} I - (\gamma+\mu)E\frac{I^*}{I} + \frac{(\delta+\mu)(\gamma+\mu)}{\gamma} I^*\\ &= \beta S^* I^* N_T + (\alpha+\mu)S^* - \beta S I N_T - (\alpha+\mu)S - \beta S^* I^* N_T \frac{S^*}{S} - (\alpha+\mu)S^*\frac{S^*}{S}\\ &\quad + \beta S^* I N_T + (\alpha+\mu)S^* + \beta S I N_T - \beta S I N_T \frac{E^*}{E} + \beta S^* I^* N_T - \beta S^* I^* N_T \frac{I}{I^*}\\ &\quad - \beta S^* I^* N_T \frac{E I^*}{E^* I} + \beta S^* I^* N_T\\ &= (\alpha+\mu)S^*\Big(2 - \frac{S}{S^*} - \frac{S^*}{S}\Big) + \beta S^* I^* N_T\Big(3 - \frac{S^*}{S} - \frac{E I^*}{E^* I} - \frac{S I E^*}{S^* I^* E}\Big).\end{aligned}$$

Using the fact that

$$\frac{S}{S^*}\frac{S^*}{S} = 1, \text{ and } \frac{S^*}{S}\frac{E I^*}{E^* I}\frac{S I E^*}{S^* I^* E} = 1$$

and the following inequality:

$$\sum_{i=1}^{i=n} x_i \geq n\left[\prod_{i=1}^{i=n} x_i\right]^{\frac{1}{n}}, \quad x_1, x_2, x_3, \cdots, x_n \geq 0, \tag{4}$$

we obtain the following inequalities:

$$2 - \frac{S^*}{S} - \frac{S}{S^*} \leq 0, \text{ and } 3 - \frac{S^*}{S} - \frac{E I^*}{E^* I} - \frac{S I E^*}{S^* I^* E} \leq 0.$$

Therefore, $\dot{Ł}_2 \leq 0$. With the help of the Lyapunov stability theorem, we deduce that $P^* = (S^*, E^*, I^*, Q^*, R^*, C^*)$ is stable.

It remains to show that $P^* = (S^*, E^*, I^*, Q^*, R^*, C^*)$ is asymptotically stable using the Lasalle invariance principle [11]. Denote

$$A_1 := 2 - \frac{S^*}{S} - \frac{S}{S^*} \quad \text{and} \quad A_2 := 3 - \frac{S^*}{S} - \frac{E I^*}{E^* I} - \frac{S I E^*}{S^* I^* E}.$$

Then one has $\dot{Ł}_2(S, E, I, Q, R, C) = 0 \Leftrightarrow A_1 = A_2 = 0$.

With the above equations, we obtain the following implications:

$$\begin{array}{lll} A_1 = 0 & \Rightarrow & S = S^*, \\ (S = S^*, A_2 = 0) & \Rightarrow & IE^* = I^*E. \end{array}$$

Finally, we get

$$\dot{\mathrm{L}}_2(S, E, I, Q, R, C) = 0 \Leftrightarrow S = S^*,\ IE^* = I^*E. \tag{5}$$

Let $e = \dfrac{E}{E^*} = \dfrac{I}{I^*}$, then $E = eE^*$ and $I = eI^*$. Replacing $S$, $I$ in the first equation of (1) at equilibrium yields

$$\mu N_T = e\beta S^* I^* N_T + (\alpha + \mu)S^* = \beta S^* I^* N_T + (\alpha + \mu)S^*.$$

Therefore, we get $e = 1$ and then $I = I^*$ and $E = E^*$. Finally,

$$\dot{\mathrm{L}}_2(S, E, I, Q, R, C) = 0 \Leftrightarrow (S = S^*, E = E^*, I = I^*, Q = Q^*, R = R^*, C = C^*).$$

Thus, the largest invariant set contained in $\{(S, E, I, Q, R, C) \big| \dot{\mathrm{L}}_2 = 0\}$ is $\{(S^*, E^*, I^*, Q^*, R^*, C^*)\}$. Then the global stability of the disease-persistence equilibrium $P^* = (S^*, E^*, I^*, Q^*, R^*, C^*)$ follows according to the Lasalle invariance principle [12].

## 6 Numerical Simulations

We validate numerical simulations for system (1). We consider four cases; two of them (Figure 1) confirming the global stability of the disease-free equilibrium $\bar{P}$ when $\mathcal{R}_0 \leq 1$. The other two tests (Figure 2) confirm the global stability of the disease-persistence equilibrium $P^*$ when $\mathcal{R}_0 > 1$.

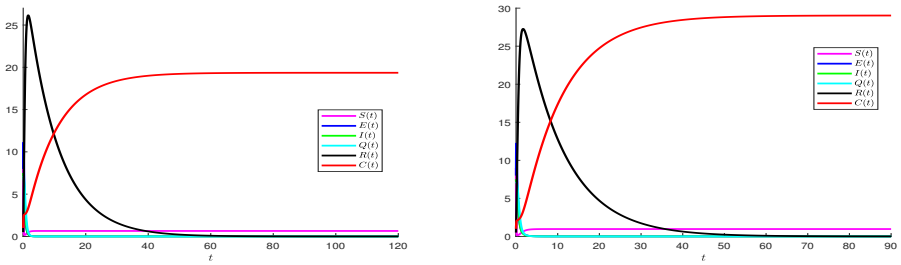


**Figure 1**: $(S(t), E(t), I(t), Q(t), R(t), C(t))$ behaviours for (left) $N_T = 20$, $\mu = 0.1$, $\beta = 0.1$, $\alpha = 3$, $\gamma = 3$, $\delta = 5$, $\lambda = 5$, $\mathcal{R}_0 = 0.49 \leq 1$ and for (right) $N_T = 30$, $\mu = 0.1$, $\beta = 0.1$, $\alpha = 3$, $\gamma = 3$, $\delta = 5$, $\lambda = 5$, $\mathcal{R}_0 = 0.74 \leq 1$.

We remark that the solution of (1) converges asymptotically to $\bar{P}$. Only susceptible and confined susceptible compartments persist, the other compartments vanish.

In this case, the solution of (1) converges asymptotically to $P^*$ and all compartments persist.

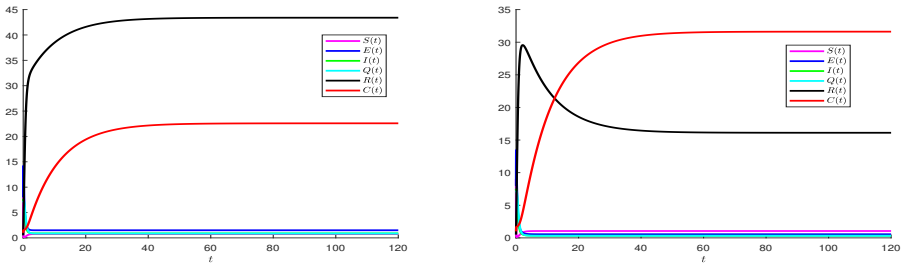


**Figure 2**: $(S(t), E(t), I(t), Q(t), R(t), C(t))$ behaviours for (left) $N_T = 70,\ \mu = 0.1,\ \beta = 0.1,\ \alpha = 3,\ \gamma = 3,\ \delta = 5,\ \lambda = 5,\ \mathcal{R}_0 = 1.73 > 1$ and for (right) $N_T = 50,\ \mu = 0.1,\ \beta = 0.1,\ \alpha = 3,\ \gamma = 3,\ \delta = 5,\ \lambda = 5,\ \mathcal{R}_0 = 1.24 > 1$.

## 7 Concluding Remarks

There is a dearth of epidemiological information on the rise of the coronavirus, which would be of critical importance to the structure and execution of auspicious, specially designated, sustainable general welfare intercessions, isolation and travel limitations. Infectious disease modelling is a tool that can be used to study the mechanisms by which diseases spread, predict the future course of the disease outbreaks and evaluate epidemic control strategies. A mathematical 6D dynamical system modelling an SEIQRC model of transmissibility of the novel Covid-19 is studied. A profound study is given. The analysis of the local and global stability of equilibrium points is presented. It is shown that the disease-persistence equilibrium is globally asymptotically stable when $\mathcal{R}_0 > 1$. However, the disease-free equilibrium is globally asymptotically stable when $\mathcal{R}_0 \leq 1$.

## Acknowledgment

The author wish to thank the Editor and the anonymous Reviewers for their helpful feedback to improve the quality of this manuscript.

## References

[1] S.A. Alharbi, A.S. Rambely and A. Othman Almatroud. Dynamic Modelling of Boosting the Immune System and Its Functions by Vitamins Intervention. *Nonlinear Dynamics and Systems Theory* **19** (2) (2019) 263–273.

[2] T. Chen, J. Rui, Q. Wang, Z. Zhao, J.A. Cui and L. Yin. A mathematical model for simulating the phase-based transmissibility of a novel coronavirus. *Infectious Diseases of Poverty* **9** (24) (2020), 8pages.

[3] O. Diekmann and J. Heesterbeek. On the definition and the computation of the basic reproduction ratio $\mathcal{R}_0$ in models for infectious diseases in heterogeneous populations. *Journal of Mathematical Biology* **28** (4) (1990) 365–382.

[4] E.F. Doungmo Goufo and G.M. Moremedi. Mathematical Analysis of a Differential Equation Modeling Charged Elements Aggregating in a Relativistic Zero-Magnetic Field. *Nonlinear Dynamics and Systems Theory* **19** (1-SI) (2019) 141–150.

[5] L. Lopez and X. Rodo. A modified SEIR model to predict the COVID-19 outbreak in Spain and Italy: simulating control scenarios and multi-scale epidemics. *Results Phys* (2021) Feb; 21: 103746.

[6] P. Van den Driessche and J. Watmough. Reproduction Numbers and Sub-Threshold Endemic Equilibria for Compartmental Models of Disease Transmission. *Math. Biosci.* **180** (2002) 29–48.

[7] C. Huang, Y. Wang, X. Li, L. Ren, J. Zhao and Y. Hu Y. *Clinical Features of Patients Infected with 2019 Novel Coronavirus in Wuhan.* China. Lancet. 2020.

[8] M. El Hajji. Modelling and optimal control for Chikungunya disease. *Theory Biosci* **140** (2021) 27–44.

[9] M. El Hajji, S. Sayari and A. Zaghdani. Mathematical analysis of an "SIR" epidemic model in a continuous reactor - deterministic and probabilistic approaches. *J. Korean Math. Soc.* **58** (1) (2021) 45–67.

[10] M. El Hajji. How can inter-specific interferences explain coexistence or confirm the competitive exclusion principle in a chemostat. *Int. J. Biomath.* **11** (8) (2018), 1850111.

[11] J.K. Hale. *Ordinary Differential Equations.* John Wiley & Sons, New York. 1969.

[12] J.P. Lasalle. *The Stability of Dynamical Systems.* SIAM, Philadelphia, 1976.

[13] Q. Li, X. Guan, P. Wu, X. Wang, L. Zhou and Y. Tong Y. Early transmission dynamics in Wuhan, China, of novel coronavirus-infected pneumonia. *N. Engl. J. Med.* **382** (13) (2020) 1–9.

[14] B. Labed, S. Kaouache and M. S. Abdelouahab. Control of a Novel Class of Uncertain Fractional-Order Hyperchaotic Systems with External Disturbances via Sliding Mode Controller. *Nonlinear Dynamics and Systems Theory* **20** (2) (2020) 203–213.

[15] M. Z. Ndii and A. K. Supriatna. Stochastic Dengue Mathematical Model in the Presence of Wolbachia: Exploring the Disease Extinction. *Nonlinear Dynamics and Systems Theory* **20** (2) (2020) 214–227.