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Flow and Heat Transfer of a Non-Newtonian Power-Law Fluid over a Non-Linearly Stretching Sheet with Thermal Radiation and Aligned Magnetic Field

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Abstract: In the present paper the effect of a non-linearly permeable stretching sheet on the solution profile in the presence of thermal radiation and aligned magnetic field has been investigated. A drive has been undertaken to thus highlight the effects of heat and mass transfer of a non-Newtonian power-law fluid over a stretching sheet when the equations are transformed into ordinary differential equations using similarity variables. The transformed equations have been solved numerically using the Runge-Kutta method coupled with the shooting technique. These results are presented graphically for various values of power-law index and for different parameters, viz the stretching parameter, suction parameter, Prandtl number radiation parameter etc.

Keywords: *MHD, non-Newtonian power law fluids; stretching sheet; thermal radiation.*

Mathematics Subject Classification (2010): 76A05, 85A30.

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1 Introduction

The study of the heat and mass transfer of a fluid over a stretching sheet has attracted many researchers these days due to its various applications in industry, for example, manufacturing of rubber sheet and plastic sheet. Fluids are used in making polymers, blowing of glass, petroleum productions, polymer extrusion, crystal growing fiber spinning and many more. Schowalter[17] was the first one who applied the boundary layer theory to the power-law pseudo-plastic fluid. After that, Schowalter and Collins [9] studied the behaviour of non-Newtonian fluid in the entry region of pipe. Crane [8] was the first to study the fluid flow past a stretching plate. Crane and Carragher [12] have studied the heat transfer on a stretching sheet instead of a plate.

After that, various researchers studied the effect of a fluid flow on a stretching sheet. Gupta and Gupta [1] studied the heat and mass transfer on a stretching sheet with suction or blowing. Vajravelu and Hadjinicolaou [7] analysed the heat transfer on a stretching sheet in the presence of dissipation and heat generation. Dandapat and Gupta [4] studied the flow and heat transfer in a viscoelastic fluid over a stretching sheet. Mahapatra and Gupta [16] have contributed on the heat transfer in the stagnation point flow towards a stretching sheet. Many researcher applied MHD flow and heat transfer over a stretching sheet due to its applications in the industries. Andersson [5] first of all studied the effect of MHD flow of a viscoelastic fluid. Andersson, Bech and Dandapat [5] studied a magneto-hydrodynamic flow of a power-law fluid over a stretching sheet. Siddheshwar and Mahabaleswar [13] studied the effects of radiation and heat source on the MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet. Cortell [14] researched on a viscous flow and heat transfer over a nonlinearly stretching sheet. Fang et al. [15] found out about a slip MHD viscous flow over a stretching sheet.

Researchers discussed several models to analyse the non-Newtonian behaviour of fluids. Among these models the power-law model gained much popularity due to its successful applications of boundary layer assumptions. Till date, the power-law fluid model is the most widely used model to describe non-Newtonian fluids behaviour. Andersson [5] was the first to study the MHD flow on the power-law fluid model.

Various researchers have discussed the power-law fluid model to study various effects on a continuously stretching sheet under different circumstances. However, all of them discussed the presence of magnetic field in transverse direction. But to the best of the authors' knowledge no one has studied the effect of aligned magnetic field on the power-law model. Zhong et. al. discussed explicit solutions of a class of linear partial difference equations with constant coefficients. Aleksandrov and Platonov [2] studied the conditions of ultimate boundedness of solutions for a class of nonlinear systems.

Based on the review of the above studies, the main objective of this paper is to analyze the effects of variable thermal conductivity on the power-law fluid flow and heat transfer over a non-linearly stretching sheet in the presence of aligned magnetic field and considering suction and radiation.

2 Formulation of the Problem

The model being considered here consists of a steady two-dimensional flow of an incompressible non-Newtonian fluid following the power law over a permeable stretching sheet. The origin is located at the slit, through which the sheet is drawn through the fluid medium. The velocity of the model is denoted by u_w.

Equations related to the power-law model for the non-Newtonian fluid, with allowance for the viscous dissipation and aligned magnetic field, are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n - \frac{\sigma(B_0 \sin\alpha)^2 u}{\rho}, \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + K\left(\frac{\partial u}{\partial y}\right)^{n+1} - \frac{\partial q_r}{\partial y}. \tag{3}$$

Here u and v are the components of fluid velocity in x and y directions, respectively. ρ, k and K represent the density of fluid, thermal conductivity and consistency coefficient, n is the power law index, B_0 is the magnetic parameter, T is the temperature of the fluid, C_p is the specific heat at constant pressure, σ is electrical conductivity, σ_s is the Stefan-Boltzmann constant and q_r is the radiative heat flux approximated by the Rosseland approximation as

$$q_r = \frac{-4\sigma_s \partial T^4}{\partial y}. \tag{4}$$

Boundary conditions for the model are

$$\begin{gathered} u = u_w(x) = cx^m + u_{sl},\ T = T_w,\ \text{when } y = 0, \\ u \longrightarrow U = bx^m,\ T \longrightarrow T_\infty,\ \text{when } y \longrightarrow \infty. \end{gathered} \tag{5}$$

Applying the boundary conditions in (5) to the model equation (2), we get

$$U\frac{dU}{dx} = \frac{-1}{\rho}\frac{\partial p}{\partial x} - \frac{\sigma(B_0 \sin\alpha)^2 u}{\rho}. \tag{6}$$

Eliminating $\frac{-1}{\rho}\frac{\partial p}{\partial x}$ from equations (2) and (6), we get

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = U\frac{\partial u}{\partial x} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n - \frac{\sigma(B_0 \sin\alpha)^2 (u-U)}{\rho}. \tag{7}$$

For further analysis we introduce similarity variable η with dimensionless variables f and θ as

$$\eta = \left[\frac{c^{2-n}}{\frac{K}{\rho}}\right]^{\frac{1}{n+1}} x^{\frac{m(2-n)-1}{n+1}}\ y\ ,\Psi(\eta) = \left(\frac{K}{\rho}\right)^{\frac{1}{n+1}} C^{\frac{2n-1}{n+1}} x^{\frac{m(2n-1)+1}{n+1}} f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T - T_w}. \tag{8}$$

With all the above similarity variables, the equations (7) and (3) reduce to

$$(f'')^{n-1} f''' + \left[\frac{m(2n-1)+1}{n+1}\right] f f'' - m\left(f'\right)^2 - H\left(\sin\alpha\right)^2 (f' - \lambda) + m\lambda^2 = 0, \tag{9}$$

$$\left(\frac{3+4R}{3}\right)\frac{\theta''}{P_R} + \left[\frac{1+m(2n-1)}{n+1}\right] f\theta' - rf'\theta + E_C\left(f''\right)^{n+1} = 0. \tag{10}$$

The transformed boundary conditions are

$$f(0) = s, f'(0) = 1, +af''(0), \theta(0) = 1 \text{ when } \eta = 0, \tag{11}$$

$$f'(\infty) \to \lambda, \theta(\infty) \to 1 \text{ when } \eta \to \infty, \tag{12}$$

where $P_R = \frac{\rho C_p}{k}\left(\left(\frac{K}{\rho}\right)^2 c^{(3m-1)(n-1)}\right)^{\frac{1}{n+1}}$ is the non-Newtonian Prandtl number and $E_C = \frac{c^2 x^{2m}}{A x^r C_p}$ is the Eckert number and $H = \sigma \frac{B_0^2}{\rho c x^{1-m}}$, s is the suction parameter, a is the constant and λ is the ratio of free stream velocity parameter and stretching parameter.

3 Solution of the Problem

To analyze the above flow model for the equations (9) and (10) along with boundary conditions (11) and (12), the fourth-order Runge-Kutta method with the shooting technique is used. With the help of the Newton-Raphson shooting method estimates for $f''(0)$and $\theta(0)$ have been done so that the equations can be integrated with the help of the fourth-order Runge-Kutta method. The process of these iterations does not stop until the boundary conditions at infinity become zero. This iteration process has been performed for each value.

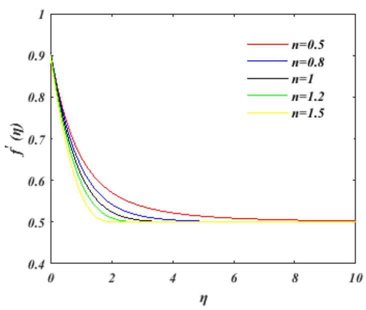
4 Results and Discussion

In this section the fourth-order Runge-Kutta method with the help of shooting method has been used to solve equations (9) and (10) for the various values of $P_r, n, m, r, R, \lambda, a, s, \alpha$ and H. Values of $f"(0)$ and $\theta(0)$ have been evaluated correctly up to 8 decimal places so that stability and accuracy of the study can be shown. It is observed from Table 1 that the numerical values of $f''(0)$ in the present paper for different values of power-law index n, when $\lambda = 0, a = 0, s = 0, s = 0, m = 1$, are in good agreement with the results obtained by Andersson and Kumaran [6], Mahmoud and Megahed [11] and Megahed [10].

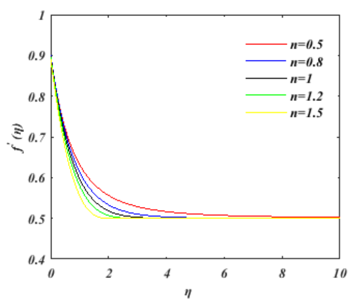
By considering the numerical solutions for the various values of power-law index in the range $0.5 \leq n \leq 1.5$, the effects of various parameters on the velocity and the temperature distributions are studied.

n	Andersson and Kumaran[24]	Mahmoud and Megahed[32]	Megahed[35]	Present Paper
0.5	1.1605	1.1604	1.1604	1.16065620
0.6	1.0951	1.0951	1.0951	1.9512419
0.7	1.545	1.544	1.544	1.054660
0.8	1.0284	1.0284	1.0284	1.02841233
0.9	1.0113	1.0112	1.0112	1.01133117
1.0	1.0000	1.000	1.00000	1.000000
1.1	0.9924	0.9922	0.9922	0.992426994
1.2	0.9874	0.9874	0.9874	0.98737207
1.3	0.9840	0.9841	0.9841	0.9840342
1.4	0.9819	0.9819	0.9819	0.9818837043
1.5	0.9806	0.9806	0.9806	0.98056261
1.6	0.9798	0.9799	0.9799	0.979825441
1.7	0.979501	0.979503	0.979503	0.975007899
1.8	0.979468	0.979467	0.979467	0.971189942
1.9	0.9796	0.9796	0.9796	0.971189943
2.0	0.9800	0.97995	0.97995	0.97912466089

Table 1: Comparison of the values of $f''(0)$ for the various values of n with the parameters' values $\lambda = 0, a = 0, s = 0, s = 0, m = 1$.



(a)



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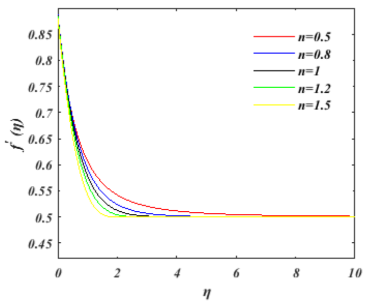


(c)

Figure 1: Figure (a), (b), (c) show the effects of $\alpha = \pi/8, \pi/4$ and $\pi/2$ on the velocity profile for different values of power-law index (η), respectively.

n	Value of $f^{'}$ and $\theta^{'}$ for different values of α with $Pr = 0.71, Ec = 0.1,$ $H = 1, R = 1, s = 0.2 = 0.2, r = 1, m = 0.5, \lambda = 0.5$					
	f''			θ'		
	$\alpha = \pi/8$	$\alpha = \pi/4$	$\alpha = \pi/2$	$\alpha = \pi/8$	$\alpha = \pi/4$	$\alpha = \pi/2$
$n = 0.5$	-0.439502	-0.506112	-0.584217	-0.565743	-0.556586	-0.546898
$n = 0.8$	-0.445340	-0.499400	-0.563085	-0.569565	-0.562002	-0.553820
$n = 1$	-0.461656	-0.510364	-0.567860	-0.568936	-0.562320	-0.555068
$n = 1.2$	-0.480821	-0.525376	-0.578055	-0.567408	-0.561607	-0.555187
$n = 1.5$	-0.510534	-0.550243	-0.597283	-0.564679	-0.559882	-0.554400

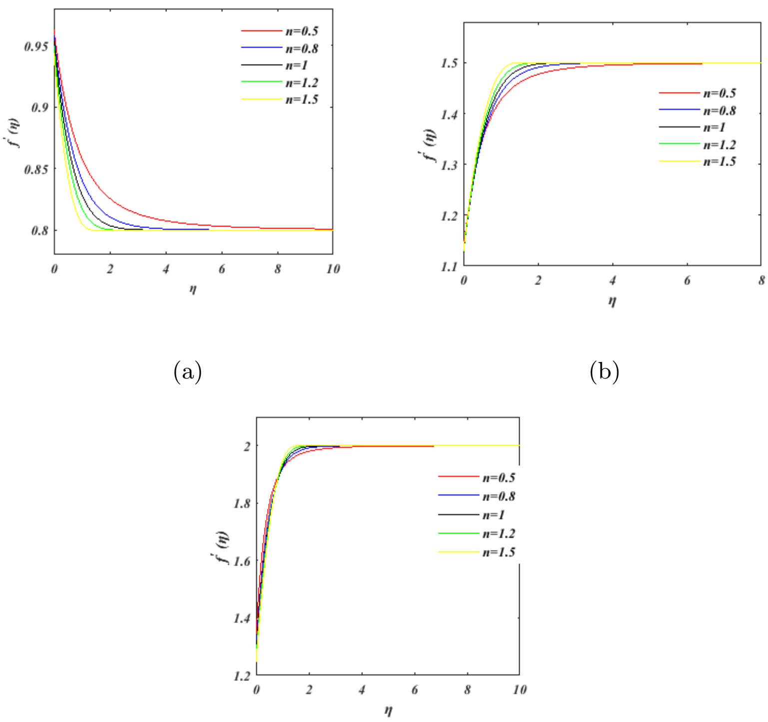
Table 2: Values of $f''(0)$ and $\theta'(0)$ for different types of fluid and different angle of magnetic field.

n	Value of $f^{'}$ and $\theta^{'}$ for different values of λ with $Pr = 0.71, Ec = 0.1,$ $H = 1, R = 1, s = 0.2, r = 1, m = 0.5, \lambda = 0.5$ and $\alpha = \pi/3$					
	f''			θ'		
	$\lambda = 0.8$	$\lambda = 1.5$	$\lambda = 2$	$\lambda = 0.8$	$\lambda = 1.5$	$\lambda = 2$
$n = 0.5$	-0.1857688	0.6706540	1.685632	-0.6306510	-0.7615877	-0.854726
$n = 0.8$	-0.2913153	0.6308961	1.440967	-0.6360154	-0.7734705	-0.856978
$n = 1.0$	-0.2289438	0.6367480	1.352774	-0.676532	-0.7807228	-0.859495
$n = 1.2$	-0.2495882	0.6429972	1.292932	-0.6385992	-0.7871893	-0.862109
$n = 1.5$	-0.2803766	0.6575240	1.232879	-0.6394543	-0.7954666	-0.865779

Table 3: Values of f' and θ' for various values of λ.

The effects of magnetic field angle α on the velocity profiles by changing the values of power law index between the range $0.5 \leq n \leq 1.5$ are shown in Figures 1(a)-1(c). It is seen that as the angle of magnetic field increases, velocity decreases for both values of the power-law index, i.e, $n < 1$ and $n \geq 1$. But it can hardly make any difference for the temperature distribution profile which is shown in Table 2.

The effects of parameter λ, which is basically the ratio of free stream velocity to the stretching parameter, on the solution profile are shown in Figures 2(a)-2(c). It is seen that with the increase in the values of λ, the velocity increases tremendously for the power-law index between $0.5 \leq n \leq 1.5$. The downfall of the temperature profile with the increase in λ is shown in Table 3.



(a) (b)

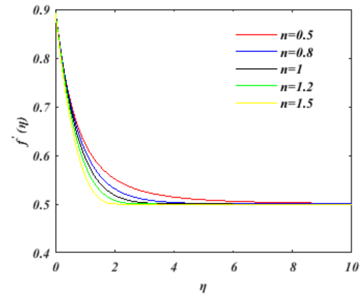
(c)

Figure 2: Figure (a), (b), (c) show the effects of $\lambda = 0.8, 1.5$ and 2 on the velocity profile for different values of power-law index (η), respectively.

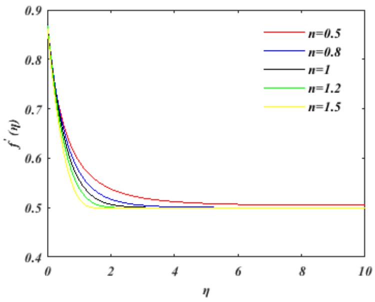
Figures 3(a)–3(c) represent the velocity profile for the values of suction parameter $s = 0, 1, 2$, respectively.

The effects of the Hartmann number H on the velocity profile and temperature profile are shown in Figures 4(a)-4(b). It is seen from the figures that the velocity and temperature decrease with the increase of H.

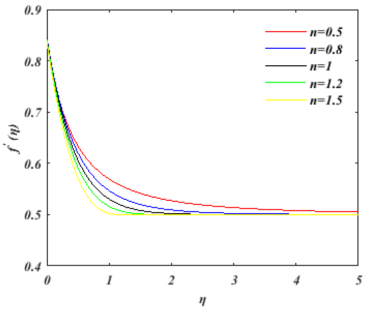
The effect of the Prandlt number Pr on the dimensionless temperature profile is illustrated in Figure 5(a). It is seen that the increase in the Prandlt number Pr decreases the temperature distribution. Figure 5(b) shows the effect of change in the radiation parameter R on the temperature profile. It is seen from the figure that the temperature increases with the increase in radiation parameter for $n = 0.5$.



(a)

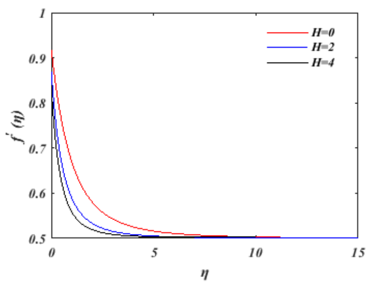


(b)

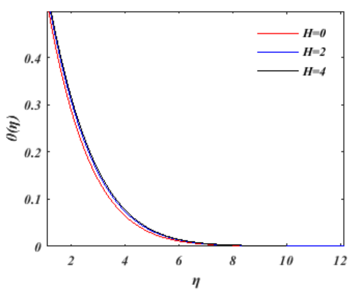


(c)

Figure 3: Figure (a), (b), (c) show the effects of $s = 0, 1$ and 2 on the velocity profile for different values of power-law index (η), respectively.

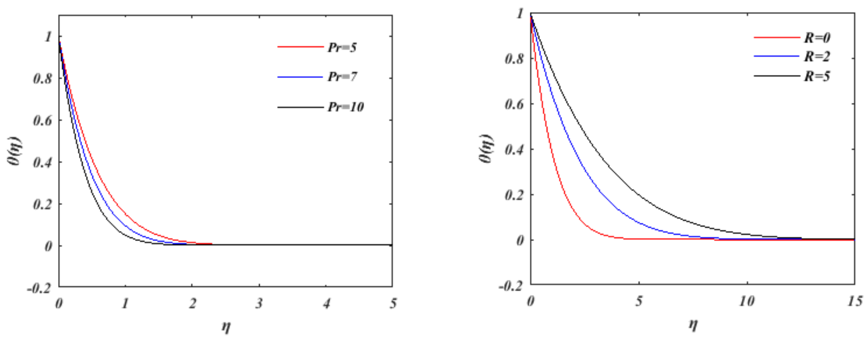


(a)



(b)

Figure 4: Figure (a), (b), (c) show the effects of different values of H on velocity profile and temperature profile, respectively.



(a) (b)

Figure 5: Figure (a), (b), (c) show the effects of different values of Pr and R on the temperature profile, respectively.

5 Conclusion

The first aim of the paper is the solution of the problem on a two dimensional flow of an incompressible non-Newtonian fluid following the power law over a permeable stretching sheet in the presence of thermal radiation. The second one is the transformation of the governing equations into a system of non-linear ordinary differential equations by using similarity transformations, which have been solved numerically using the fourth-order Runge-Kutta method coupled with the shooting technique. During the study the following observations have been achieved:

- Increase in the angle of magnetic field decreases the velocity of the fluid.
- Increase in the stretching parameter increases the velocity distribution but decreases the temperature distribution.
- As suction increases in the sheet, both velocity and temperature decrease.
- Temperature decreases with the increase in the Prandlt number.
- As the effect of magnetic field increases, the velocity decreases.
- The radiation parameter increases with the increase of temperature.

Nomenclature

B_0= magnetic field intensity
c= stretching sheet parameter
C_p= specific heat at constant pressure
K= consistency coefficient
k'= permeability of the medium
k= thermal conductivity
H= Hartmann number $\frac{\sigma B_0^2}{c\rho}$
Pr= Prandtl number
p_r= radiative heat flux
q_r= rate of heat transfer
R= radiation parameter($= (4\sigma_s T_\infty^3)/Kk$)
s=heat source/sink parameter
T= fluid temperature
T_∞= free stream temperature
T_w= temperature of stretching sheet
u, v= velocity components along x and y axes, respectively
$u_w(x)$=velocity of stretching sheet
$U(x)$=free stream velocity($= bx^m$)
x, y= Cartesian coordinates along x and y axes, respectively.

Greek symbols

η= similarity variable
Ψ stream function
σ = electrical conductivity
ν=kinematic viscosity
μ =coefficient of viscosity
σ_s=Stefan-Boltzmann constant
Λ = ratio of free stream velocity parameter to stretching sheet parameter.

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