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Model-Based Iterative Learning Control for the Trajectory Tracking of Disturbed Robot Manipulators

Chems Eddine Boudjedir [1*], Djamel Boukhetala [1] and Mohamed Bouri [2]

[1] *Laboratory of Process Control (LCP), Ecole Nationale Polytechnique (ENP), 10 Rue des Freres OUDEK, El-Harrach 16200, Algiers, Algeria.*
[2] *Labs of Biorobotics (BIOROB) and Translational Neural Engineering (TNE), Ecole Polytechnique Federale de Lausanne (EPFL), Station 9, Lausanne CH-1015, Switzerland.*

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Abstract: This paper proposes a model-based iterative learning control (ILC) for the trajectory tracking problem of robot manipulators performing repetitive tasks and subjected to external disturbances. The proposed scheme consists of a model-based controller to compensate as much as possible the coupled robot dynamics, a PD-type ILC to improve the tracking performances through the repetitive trajectory as well as a robust term to reject the effects of the disturbances. The convergence analysis is driven using Lyapunov theory. It is shown that the tracking error converges to zero when the iteration number increases to infinity. Simulations are performed on the parallel Delta robot to demonstrate the feasibility of the proposed approach and to highlight its tracking performances. A comparative study between the proposed ILC, the conventional PID controller, and the traditional PD plus PD-type ILC is conducted to point out the effectiveness of the model-based ILC.

Keywords: *iterative learning control; model-based control; Lyapunov theory; delta robot.*

Mathematics Subject Classification (2010): 37B25, 70E60, 93D09.

* Corresponding author: mailto:chemseddine.boudjedir@g.enp.edu.dz

1 Introduction

Over the years, many robot manipulators have been invented to satisfy industrial needs, for example, the Stewart platform [1] and the Delta robot [2]. The conventional control methods like PD/PID are often used to achieve the required tasks of these robots. However, robot manipulators are subject to external disturbances and their dynamics are characterised by strong coupling between the joints, which has an important effect at high dynamic movements. Therefore, PD/PID controllers are not satisfactory for applications that require high tracking accuracy at high cadence. This is due to the fact that the controller gains are selected without considering the coupling effects and external disturbances. To overcome these problems, many advanced controllers have been successfully implemented. One of the interesting techniques is the model-based controller [3]. This centralized strategy integrates the nonlinear robot dynamics in the control design to stabilise and to compensate a large part of the coupled dynamics. However, the lack of very accurate knowledge of the system may decrease the tracking performance, especially for robot manipulators that move at high speed. Other advanced controllers have been developed by considering in their objective the disturbances and coupling effects, for instance, the nonlinear PD plus sliding mode control [4], robust H-infinity control [5, 6], and neural network controllers [7, 8].

Robot manipulators are usually used for repetitive tasks such as laser cutting [9] and pick and place operations [10], where the desired trajectory is repeated over a finite time interval. Unfortunately, the most well-known controllers are not able to benefit from the task repeatability which yields the same performance without improvement. In order to exploit these repetitions, the idea of iterative learning control approach has emerged. The ILC controller takes into account the information of the previous cycles in order to improve the tracking performances of the current cycle.

In the early 1990s, Arimoto proposed a series of learning laws such as the PD-type and PID-type ILC [11]. After that, the ILC has been extensively studied, where several ILC schemes for robot manipulators have been proposed [12,13]. Others strategies, based on adaptive and robust learning have been developed to overcome parametric and non-parametric uncertainties effects, they are: the adaptive ILC [14], adaptive switching PD control strategy [15], and robust ILC [16]. For a comprehensive review on the ILC, the readers may refer to the survey given in [17] and [18]. It is noted that the ILC has been successfully applied in many areas such as robotics [19, 20] and biological systems [21].

The main contribution of this work is to develop a model-based ILC for the trajectory tracking problem of perturbed robot manipulators performing repetitive tasks. In contrary to the traditional ILC [11], the proposed controller allows to compensate the unknown uncertainties of robot manipulators as well as the external disturbances. Moreover, the model-based ILC is more practical than the adaptive ILC [22], [15] that assumes the dynamic model can be expressed by a pre-multiplication of two separate knowing matrices and unknowing vector, which can not be the case for a complex robot like our application of the "Delta robot". Thus, the proposed control law consists of two terms. The first item is a model-based controller represented by the pre-multiplication between the PD controller and the inertia matrix. The second item is a learning control scheme that consists of a PD-type ILC with an additional robust term. Compared to the existing works related to the application of ILC, the proposed controller can be applied to uncertain nonlinear dynamic, unlike [16, 23, 24], where the controller is designed to discrete-time linear systems. Unlike [13, 15, 25], where the ILC schemes are specifically developed for repetitive disturbances, the controller in this work can deal with nonrepetitive disturbances. The stability of the proposed control is proved using the Lyapunov method. It is shown that the tracking error converges to zero in a finite time interval when the number of iterations approaches infinity. To demonstrate the feasibility and the performances of the proposed controller, simulations have been performed on a parallel Delta robot and followed by a comparative study between the proposed controller, the conventional PID controller and the traditional PD plus PD-type ILC controller.

Throughout this paper, we use the notation $\lambda_{min}(A)$ and $\lambda_{max}(A)$ to indicate the minimum and the maximum eigenvalue of matrix A, and for any $x \in \mathbb{R}^n$, the norm of vector x is defined as $\|x\|=\sqrt{x^T x}$, while the norm of matrix A is defined as follows: $\|A\|=\sqrt{\lambda_{max}(A^T A)}$.

2 Problem Formulation

Consider the actual dynamic model for a rigid robot with n-degrees of freedom described by

$$M(q_k)\ddot{q}_k + C(q_k, \dot{q}_k)\dot{q}_k + G(q_k) + w_k(t) = \tau_k, \tag{1}$$

where $q_k \in \mathbb{R}^n$ is the generalized joint vector, $M(q_k)$ is the inertia matrix, $C(q_k, \dot{q}_k)\dot{q}_k$ is a vector resulting from Coriolis and centrifugal forces, $G(q_k)$ is the gravity torque vector, τ_k is the control input vector containing the torques to be applied at each joint and $w_k(t)$ is the vector containing external disturbances. The index k denotes the iteration number.

The actual robot dynamics (1) can be written using the nominal model as follows:

$$M_n(q_k)\ddot{q}_k + C_n(q_k, \dot{q}_k)\dot{q}_k + G_n(q_k) + d_k(t) = \tau_k, \tag{2}$$

where

$$d_k(t) = \Delta M(q_k)\ddot{q}_k + \Delta C(q_k, \dot{q}_k)\dot{q}_k + \Delta G(q_k) + w_k(t). \tag{3}$$

Our objective is to design an iterative control law τ_k, which allows the robot to track any given trajectory q_d as k tends to infinity, i.e., $\lim_{k\to\infty} \tilde{q}_k(t) = q_d(t) - q_k(t) = 0$, $\lim_{k\to\infty} \dot{\tilde{q}}_k(t) = \dot{q}_d(t) - \dot{q}_k(t) = 0$, $\forall t \in [0, T]$.

The dynamic motion equation (2) has the following fundamental properties, which facilitate the convergence analysis of the proposed control law [22]:

(P1) The inertia matrix $M_n(q_k)$ is symmetric, positive and satisfies

$$0 < \alpha \le \|M_n(q_k)\| \le \beta,$$

where α, and β are known positive constants.

(P2) There exists a positive constant K_M such that the inertia matrix $M_n(q_k)$ is globally Lipschitz continuous in its arguments

$$\|M_n(q_{k+1}) - M_n(q_k)\| < K_{M_n}\|q_{k+1} - q_k\|.$$

(P3) $G_n(q_k)$ is globally Lipschitz continuous in its arguments

$$\|G_n(q_{k+1}) - G_n(q_k)\| \le K_g\|q_{k+1} - q_k\|,$$

where K_g is a known positive constant.

(P4) The following upper bounds are valid:

$$\|C_n(q_k, \dot{q}_k)\| \le K_{c1}\|\dot{q}_k\|, \quad \|G_n(q_k)\| \le K_G, \quad \forall q_k, \dot{q}_k \in \mathbb{R}^n,$$

where K_{c1} and K_G are known positive constants.

(P5) For robots having exclusively revolute joints, there exist constants $K_{c1} > 0$ and $K_{c2} > 0$ such that

$$\|C_n(q_{k+1}, \dot{q}_{k+1})\dot{q}_{k+1} - C_n(q_k, \dot{q}_k)\dot{q}_{k+1}\| \leq K_{c1}\|\dot{q}_{k+1} - \dot{q}_k\|\|\dot{q}_{k+1}\| + K_{c2}\|q_{k+1} - q_k\|\|\dot{q}_{k+1}\|^2.$$

The following assumptions are made:

(A1) The reference trajectory and its first and second time-derivatives, namely, q_d, $\dot{q}_d$, and $\ddot{q}_d$ are bounded $\forall t \in [0, T]$ and $\forall k \in Z_+$.

(A2) The resetting condition is satisfied:

$$q_k(0) = q_d(0), \quad \dot{q}_k(0) = \dot{q}_d(0), \quad \forall k \in Z_+.$$

(A3) The robot velocity is bounded by a known constant V_m such that

$$\|\dot{q}_k\| \leq V_m.$$

(A4) The external disturbances and the model uncertainty are bounded by a known constant l_d such that

$$\|d_k(t)\| \leq l_d.$$

In this paper, the following lemma is used.

Lemma [26]: *The inertia matrix $M_n(q_k)$ has the following property:*

$$\|M_n(q_{k+1})^{-1} - M_n(q_k)^{-1}\| \leq K_{M_n}\alpha^{-2}\|q_{k+1} - q_k\|.$$

Proof.

$$M_n(q_{k+1})^{-1} - M_n(q_k)^{-1} = -M_n(q_{k+1})^{-1}(M_n(q_{k+1}) - M_n(q_k))M(q_k)^{-1}.$$

From (P1) and (P2) we can obtain

$$\|M_n(q_{k+1})^{-1} - M_n(q_k)^{-1}\| \leq K_{M_n}\alpha^{-2}\|q_{k+1} - q_k\|.$$

3 Iterative Learning Control

3.1 Controller design

The model-based ILC is given below

$$\tau_k = M_n(q_k)[K_p\tilde{q}_k + K_d\dot{\tilde{q}}_k + u_k], \tag{4}$$

and the ILC expression is given by

$$u_{k+1} = u_k + \Lambda\tilde{q}_k + \Gamma\dot{\tilde{q}}_k + \mu sgn(\tilde{z}_k), \tag{5}$$

where the variables $\tilde{z}_k$ and z_k are defined as

$$\tilde{z}_k = z_{k+1} - z_k, \tag{6}$$

$$z_k(t) = \dot{\tilde{q}}_k(t) + \zeta\tilde{q}_k(t). \tag{7}$$

K_p, K_d, Γ, and Λ are diagonal matrices, and μ, ζ are positive constants.

The scheme of the proposed controller is illustrated in Fig. 1, where x_d represents the desired trajectory in the task space and the IGM indicates the inverse geometric model.

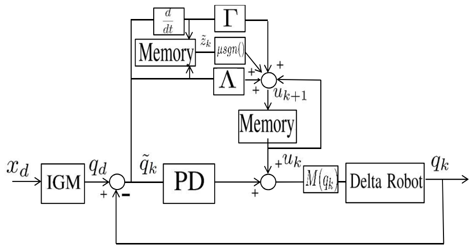


Figure 1: The proposed control scheme.

3.2 Convergence analysis

To simplify the notation, let

$$M_n(q_k) = M_{n,k}, C_n(q_k, \dot{q}_k) = C_{n,k}, G_n(q_k) = G_{n,k}, d_k(t) = d_k.$$

In the k-th iteration, equation (2) can be rewritten as

$$\ddot{q}_k = k_p\tilde{q}_k + k_d\dot{\tilde{q}}_k + u_k - M_{n,k}^{-1}C_{n,k}\dot{q}_k - M_{n,k}^{-1}G_{n,k} - M_{n,k}^{-1}d_k. \tag{8}$$

Similarly, in the (k+1)-th iteration, we have

$$\begin{aligned} \ddot{q}_{k+1} = \quad & k_p\tilde{q}_{k+1} + k_d\dot{\tilde{q}}_{k+1} + u_{k+1} - M_{n,k+1}^{-1}C_{n,k+1}\dot{q}_{k+1} \\ & -M_{n,k+1}^{-1}G_{n,k+1} - M_{n,k+1}^{-1}d_{k+1}. \end{aligned} \tag{9}$$

For the purpose of convergence proof, we assume that

$$K_p = \zeta K_d \quad \text{and} \quad \Lambda = \zeta\Gamma. \tag{10}$$

We also define the variable δq_k as

$$\delta q_k = \tilde{q}_{k+1} - \tilde{q}_k. \tag{11}$$

Theorem. *Consider the system (2) under assumptions (A1-A4), properties (P1-P5), and control law (4). Then the position and velocity tracking errors converge to zero as k approaches infinity over a finite-time interval* $[0, T]$*, i.e.,* $\lim_{k\to\infty} q_k(t) = q_d(t)$ *and* $\lim_{k\to\infty} \dot{q}_k(t) = \dot{q}_d(t)$, $\forall t \in [0, T]$*, if the gains of the controller are selected as:*

$$2\lambda_{min}(K_d - \zeta I) \geq \lambda_{max}(\Gamma) - 2\zeta + \frac{a_1 + a_2}{\zeta} \geq 0, \tag{12}$$

$$2\lambda_{min}(K_d - \zeta I) \geq \lambda_{max}(\Gamma) + a_3 \geq 0, \tag{13}$$

$$4AB \geq C^2, \tag{14}$$

$$\mu - 2l_d\alpha^{-1} > \gamma, \tag{15}$$

where

$$A = \zeta^2\lambda_{max}(\Gamma) - 2\zeta^2\lambda_{min}(K_d - \zeta I) - 2\zeta^3 + \zeta(a_1 + a_2), \tag{16}$$

$$B = \lambda_{max}(\Gamma) - 2\lambda_{min}(K_d - \zeta I) + a_3, \tag{17}$$

$$C = a_1 + a_2 + \zeta a_3, \tag{18}$$

and

$$a_1 = 2(\alpha^{-1}K_g + K_M\alpha^{-2}K_G), \tag{19}$$

$$a_2 = 2(\alpha^{-1}K_{c2}V_m^2 + \alpha^{-2}K_M K_{c1}V_m^2), \tag{20}$$

$$a_3 = 4\alpha^{-1}K_{c1}V_m, \tag{21}$$

γ *is a positive constant.*

***Proof*:** Consider the following Lyapunov function:

$$V_k(t) = \int_0^t z_k^T \Gamma z_k d\sigma. \tag{22}$$

Hence

$$\Delta V_k = V_{k+1} - V_k = \int_0^t z_{k+1}^T \Gamma z_{k+1} d\sigma - \int_0^t z_k^T \Gamma z_k d\sigma = \int_0^t \tilde{z}_k^T \Gamma \tilde{z}_k + 2\tilde{z}_k^T \Gamma z_k d\sigma. \tag{23}$$

We have $\ddot{\tilde{q}}_{k+1} - \ddot{\tilde{q}}_k = \ddot{q}_d - \ddot{q}_{k+1} - \ddot{q}_d + \ddot{q}_k$. By subtracting (9) from (8), we obtain

$$\begin{aligned} \ddot{\tilde{q}}_{k+1} - \ddot{\tilde{q}}_k = & -K_p(\tilde{q}_{k+1} - \tilde{q}_k) - K_d(\dot{\tilde{q}}_{k+1} - \dot{\tilde{q}}_k) - u_{k+1} + u_k \\ & + M_{n,k+1}^{-1}G_{n,k+1} - M_{n,k}^{-1}G_k + M_{n,k+1}^{-1}C_{n,k+1}\dot{q}_{k+1} \\ & - M_{n,k}^{-1}C_{n,k}\dot{q}_k + M_{n,k+1}^{-1}d_{k+1} - M_{n,k}^{-1}d_k. \end{aligned} \tag{24}$$

By combining the equations (5), (6), (7), (10) and (11) we get

$$\begin{gathered} \dot{\tilde{z}}_k + (K_d - \zeta I)\tilde{z}_k + \zeta^2 \delta q_k - (M_{n,k+1}^{-1}G_{n,k+1} - M_{n,k}^{-1}G_{n,k}) - \\ (M_{n,k+1}^{-1}C_{n,k+1}\dot{q}_{k+1} - M_{n,k}^{-1}C_{n,k}\dot{q}_k) - (M_{n,k+1}^{-1}d_{k+1} - M_{n,k}^{-1}d_k) + \mu sgn(\tilde{z}_k) = -\Gamma z_k. \end{gathered} \tag{25}$$

Replacing (25) in (23) gives us

$$\begin{aligned} \Delta V_k = \quad & \int_0^t \tilde{z}_k^T \Gamma \tilde{z}_k - 2\tilde{z}_k^T \dot{\tilde{z}}_k - 2\tilde{z}_k^T (K_d - \zeta I)\tilde{z}_k - \\ & 2\zeta^2 \tilde{z}_k^T \delta q_k + 2\tilde{z}_k^T (M_{n,k+1}^{-1}G_{n,k+1} - M_{n,k}^{-1}G_{n,k}) + \\ & 2\tilde{z}_k^T (M_{n,k+1}^{-1}C_{n,k+1}\dot{q}_{k+1} - M_{n,k}^{-1}C_{n,k}\dot{q}_k) + \\ & 2\tilde{z}_k^T (M_{n,k+1}^{-1}d_{k+1} - M_{n,k}^{-1}d_k - \mu sgn(\tilde{z}_k)) d\sigma. \end{aligned} \tag{26}$$

Therefore

$$\begin{aligned} \Delta V_k = & \int_0^t \tilde{z}_k^T \Gamma \tilde{z}_k - 2\tilde{z}_k^T \dot{\tilde{z}}_k - 2\tilde{z}_k^T (K_d - \zeta I)\tilde{z}_k - \\ & 2\zeta^2 \tilde{z}_k^T \delta q_k + 2\tilde{z}_k^T [M_{n,k+1}^{-1}(G_{n,k+1} - G_{n,k}) + \\ & (M_{n,k+1}^{-1} - M_{n,k}^{-1})G_{n,k}] + 2\tilde{z}_k^T [M_{n,k+1}^{-1}((C_{n,k+1} - C_{n,k})\dot{q}_{k+1} \\ & + C_{n,k}(\dot{q}_{k+1} - \dot{q}_k)) + (M_{n,k+1}^{-1} - M_{n,k}^{-1})C_{n,k}\dot{q}_k] + \\ & 2\tilde{z}_k^T (M_{n,k+1}^{-1}d_{k+1} - M_{n,k}^{-1}d_k - \mu sgn(\tilde{z}_k)) d\sigma. \end{aligned} \tag{27}$$

With assumption (A4) and property (P1) we can obtain $\tilde{z}_k^T (M_{n,k+1}^{-1}d_{k+1} - M_{n,k}^{-1}d_k) \leq \|\tilde{z}_k^T\|(2l_d\alpha^{-1})$, thus

$$\int_0^t \tilde{z}_k^T (M_{n,k+1}^{-1}d_{k+1} - M_{n,k}^{-1}d_k - \mu sgn(\tilde{z}_k)) d\sigma \leq \int_0^t \|\tilde{z}_k^T\|(2l_d\alpha^{-1} - \mu) d\sigma. \tag{28}$$

Properties (P1-P5), (28) and assumption (A3) lead to

$$\begin{aligned}\Delta V_k \leq \quad & \int_0^t \tilde{z}_k^T \Gamma \tilde{z}_k - 2\tilde{z}_k^T \dot{\tilde{z}}_k - 2\tilde{z}_k^T (K_d - \zeta I)\tilde{z}_k - \\ & 2\zeta^2 \tilde{z}_k^T \delta q_k + 2\|\tilde{z}_k^T\|(\alpha^{-1}K_g + K_M \alpha^{-2} K_G)\|\delta q_k\| + \\ & 2\|\tilde{z}_k^T\|(\alpha^{-1}K_{c2}V_m^2 + \alpha^{-2}K_M K_{c1} V_m^2)\|\delta q_k\| + \\ & 2\|\tilde{z}_k^T\|(2\alpha^{-1}K_{c1}V_m)\|\delta \dot{q}_k\| + 2\|\tilde{z}_k^T\|(2l_d\alpha^{-1} - \mu)d\sigma.\end{aligned} \tag{29}$$

By replacing (7), (19), (20) and (21) in (29) we obtain

$$\begin{aligned}\Delta V_k \leq \quad & \int_0^t \delta\dot{q}_k^T \Gamma \delta\dot{q}_k + \zeta^2 \delta q_k^T \Gamma \delta q_k + 2\zeta \delta\dot{q}_k^T \Gamma \delta q_k - 2\tilde{z}^T \dot{\tilde{z}}_k - \\ & 2\zeta^2 \delta q_k^T (K_d - \zeta I)\delta q_k - 2\delta\dot{q}_k^T (K_d - \zeta I)\delta \dot{q}_k - \\ & 4\zeta \delta\dot{q}_k^T (K_d - \zeta I)\delta q_k - 2\zeta^2 \delta\dot{q}_k^T \delta q_k - 2\zeta^3 \|\delta q_k\|^2 + \\ & \zeta a_1 \|\delta q_k\|^2 + a_1 \|\delta q_k\|\|\delta\dot{q}_k\| + \zeta a_2 \|\delta q_k\|^2 + \\ & a_2 \|\delta q_k\|\|\delta\dot{q}_k\| + a_3 \|\delta\dot{q}_k\|^2 + \zeta a_3 \|\delta q_k\|\|\delta\dot{q}_k\| + \\ & 2\|\tilde{z}_k^T\|(2l_d\alpha^{-1} - \mu)d\sigma.\end{aligned} \tag{30}$$

Using assumption (A2) one can get

$$\begin{aligned}\Delta V_k \leq \quad & -\|\tilde{z}_k\|^2 - \zeta^2 \|\delta q_k\|^2 - \zeta \delta q_k^T (2K_d - 2\zeta I - \Gamma)\delta q_k + \\ & \int_0^t \delta\dot{q}_k^T \Gamma \delta\dot{q}_k + \zeta^2 \delta q_k^T \Gamma \delta q_k - 2\zeta^3 \|\delta q_k\|^2 - \\ & 2\zeta^2 \delta q_k^T (K_d - \zeta I)\delta q_k - 2\delta\dot{q}_k (K_d - \zeta I)\delta\dot{q}_k + \\ & \zeta a_1 \|\delta q_k\|^2 + a_1 \|\delta q_k\|\|\delta\dot{q}_k\| + \zeta a_2 \|\delta q_k\|^2 + \\ & a_2 \|\delta q_k\|\|\delta\dot{q}_k\| + a_3 \|\delta\dot{q}_k\|^2 + \zeta a_3 \|\delta q_k\|\|\delta\dot{q}_k\| + \\ & 2\|\tilde{z}_k^T\|(2l_d\alpha^{-1} - \mu)d\sigma.\end{aligned} \tag{31}$$

Hence

$$\begin{aligned}\Delta V_k \leq \quad & -\|\tilde{z}_k\|^2 - \zeta^2 \|\delta q_k\|^2 - \zeta \lambda_{min}(2K_d - 2\zeta I - \Gamma)\|\delta q_k\|^2 \\ & + \int_0^t [\lambda_{max}(\Gamma) - 2\lambda_{min}(K_d - \zeta I) + a_3]\|\delta\dot{q}_k\|^2 + \\ & [\zeta^2 \lambda_{max}(\Gamma) - 2\zeta^3 - 2\zeta^2 \lambda_{min}(K_d - \zeta I) \\ & + \zeta(a_1 + a_2)]\|\delta q_k\|^2 + [a_1 + a_2 + \zeta a_3]\|\delta q_k\|\|\delta\dot{q}_k\| + \\ & 2\|\tilde{z}_k^T\|(2l_d\alpha^{-1} - \mu)d\sigma.\end{aligned} \tag{32}$$

Using (16), (17), and (18) we obtain

$$\begin{aligned}\Delta V_k \leq \quad & -\|\tilde{z}_k\|^2 - \zeta^2 \|\delta q_k\|^2 - \zeta \lambda_{min}(2K_d - 2\zeta I - \Gamma)\|\delta q_k\|^2 \\ & + \int_0^t A\|\delta q_k\|^2 + B\|\delta\dot{q}_k\|^2 + C\|\delta q_k\|\|\delta\dot{q}_k\| + 2\|\tilde{z}_k^T\|(2l_d\alpha^{-1} - \mu)d\sigma.\end{aligned} \tag{33}$$

Hence, from (15) we can get

$$\begin{aligned}\Delta V_k < \quad & -\|\tilde{z}_k\|^2 - \zeta^2 \|\delta q_k\|^2 - \zeta \lambda_{min}(2K_d - 2\zeta I - \Gamma)\|\delta q_k\|^2 \\ & + \int_0^t A(\|\delta q_k\| + \frac{C}{2A}\|\delta\dot{q}_k\|)^2 + (B - \frac{C^2}{4A})\|\delta\dot{q}_k\|^2 - 2\gamma\|\tilde{z}_k^T\|d\sigma.\end{aligned} \tag{34}$$

From (12), (13), (14) and (15) we can get

$$\Delta V_k < 0, \quad i.e., \quad V_{k+1} < V_k. \tag{35}$$

From (35) we conclude that when k tends to infinity, V_k tends to zero, which implies that $z_k \to 0$, and from the definition of z_k (7), we can obtain

$$\lim_{k\to\infty} \tilde{q}_k(t) = \lim_{k\to\infty} \dot{\tilde{q}}_k = 0, \ \forall t \in [0, T]. \tag{36}$$

Remark: It is worth noting that the sign function used in the proposed control (5) might lead to the chattering phenomenon in the control input. In order to reduce the effects of this phenomenon in practical applications, saturation function can be introduced instead of the sign function. As a consequence, the tracking error converges to a domain around zero with a smooth control signal.

4 Simulation

In this section, we present the simulation results obtained by applying the model-based ILC on the parallel Delta robot described by Fig. 2. Delta robot is a very fast robot designed to achieve high precision for high dynamic pick and place operations, where the traditional controllers can fail to deal with this dynamic and to reject the external disturbances.

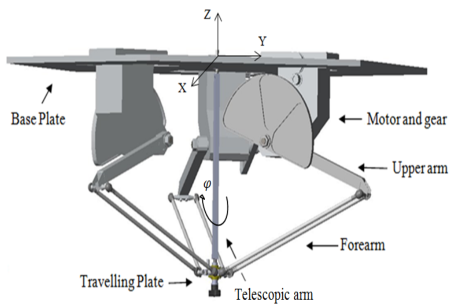


Figure 2: The Delta robot.

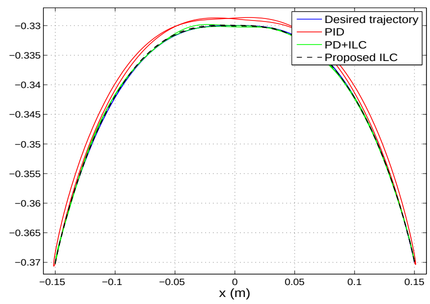


Figure 3: The trajectory tracking in the task space.

The matrices of the robot are given as follows:

$$M(q) = I_b + m_{nt} J^T J, \quad C(q, \dot{q}) = J^T m_{nt} \dot{J}, \quad G(q) = -\tau_{Gn} - \tau_{Gb},$$

where τ_{Gn} is the torque produced by the inertial force, τ_{Gb} is the torque produced by the gravitational force of the arms, J represents the Jacobian matrix and $\dot{J}$ is its time derivative, m_{nt} represents the total mass which is the sum of the travelling plate mass, the mass of the payload and the 3 reported masses contributed each of the 3 forearms. For the detailed expressions of the Jacobian, τ_{Gn}, τ_{Gb} and m_{nt}, please, refer to [27]. The geometrical and dynamic parameters of the Delta robot are described in Table 1. The constants are given as follows: $K_{c1} = 0.44\ kgm^2$, $K_{c2} = 2.675\ kgm^2$, $K_g = 0.354\ kg.m^2/s^2$, $K_G = 0.442\ kg.m^2/s^2$, $V_m = 5\ rad/s$, $\alpha = 0.3\ kgm^2$, $K_M = 0.09\ kgm^2$.
The modelling errors are set as follows: $\Delta M(q_k) = 0.1 * M(q_k)$, $\Delta C(q_k, \dot{q}_k) = 0.1 * C(q_k, \dot{q}_k)$, $\Delta G(q_k) = 0.1 * G(q_k)$. Whereas, the disturbances are assumed to be time-varying and also varying from iteration to iteration as follows: $d1(t)$=$d2(t)$=$d3(t)$=$0.2.rand(k)sin(2\pi t)$ (in Newton meters), where $rand(k)$ is a random function taking its values between 0 and 1.

Table 1: Geometric and dynamic parameters.

Parameter	Value
Length of the upper arm	0.380 m
Length of the forearm	0.205 m
Mass of the travelling plate	0.042 kg
Mass of the upper arm	0.098 kg
Masses of the forearms	0.028 kg
Mass of the elbow	0.015 kg

The desired trajectory used along the x-axis and the z-axis is a polynomial of degree five with an initial and final velocity and acceleration equal to zero. Its expression is given by

$$x(t) = x_i + (x_f - x_i)\Big(6\frac{t}{t_f}^5 - 15\frac{t}{t_f}^4 + 10\frac{t}{t_f}^3\Big), \tag{37}$$

where x_i and x_f are the initial and final positions, and t_f is the duration of the movement.

To evaluate the performance of the controllers, the Root Mean Square Error (RMSE) criteria and the Maximum Absolute Error (MaxAE) criteria are used. Their expressions are given as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - y_{d_i})^2}, \tag{38}$$

$$MaxAE = Max(|y_i - y_{d_i}|), \tag{39}$$

where y_d is the desired trajectory, y_i is the actual response, and n is the total number of samples in one iteration. The controller gains matrices were selected so that the minimum performance criteria specified by the RMSE and the MaxAE are obtained after 90 iterations. The proposed controller gains were set to: $K_p = diag\{600\}$, $K_d = diag\{40\}$, $\Lambda = diag\{18.3\}$, $\Gamma = diag\{1.22\}$, and $\zeta = 15$, while the PID controller gains were selected as: $K_{p(PID)} = diag\{12\}$, $K_{d(PID)} = diag\{0.18\}$, $K_{I(PID)} = diag\{2\}$, and the PD plus ILC controller gains were chosen as: $K_{p(PD-ILC)} = diag\{8\}$, $K_{d(PD-ILC)} = diag\{0.08\}$, $\Lambda_{PD-ILC} = diag\{0.5\}$, $\Gamma_{PD-ILC} = diag\{0.02\}$. We give the simulation study in two cases.

Case 1: The desired trajectory starts from the initial position $(-0.15, 0, -0.37)$m to the final position $(0.15, 0, -0.37)$m with a height of transit equal to 0.04 m, then returns to the initial position during 0.4 second.

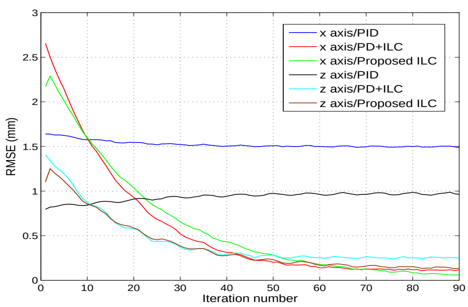


Figure 4: The RMSE along the iteration axis.

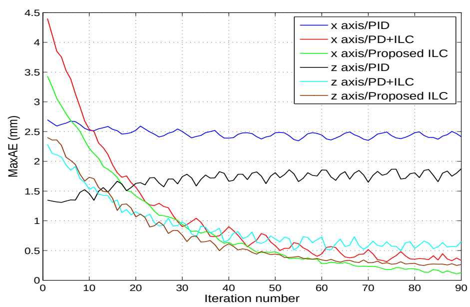
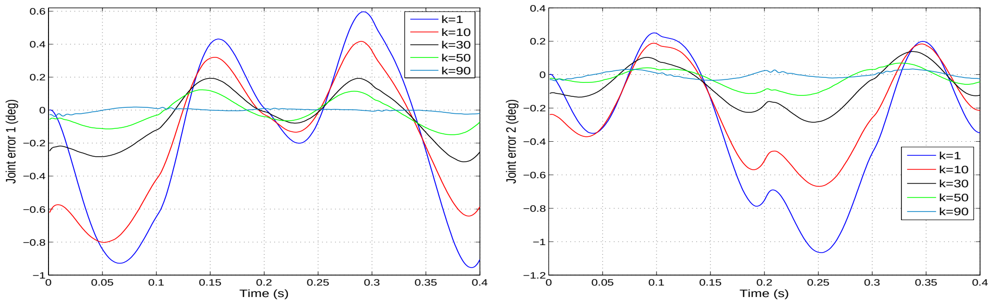
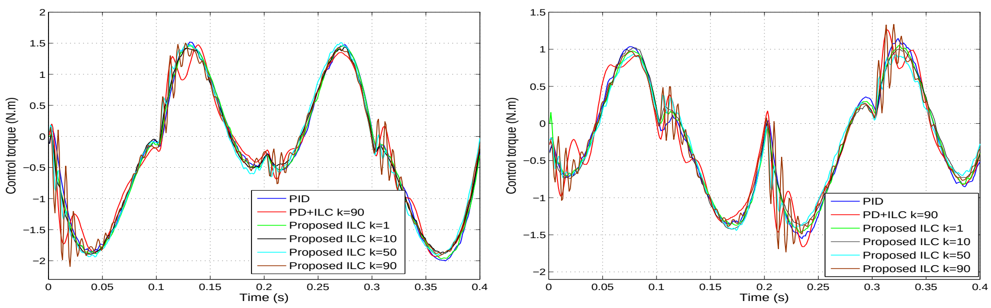


Figure 5: The MaxAE along the iteration axis.



(a) (b)

Figure 6: Tracking error for iteration k=1,10,30,50,90. (a) joint 1, (b) joint 2.



(a) (b)

Figure 7: Control torque for iteration k=1,10,30,50,90. (a) joint 1, (b) joint 2.

Figure 3 presents the trajectory tracking in the operational space after 90 iterations under the proposed controller, the PID controller and the PD plus PD-type ILC. Fig. 4 and Fig. 5 indicate the progress of the RMSE and the MaxAE, respectively, through the iterations. It can be observed that the tracking performance improves from iteration to iteration, where, for instance, the RMSE decreased from 2.17 mm at the first iteration along the x-axis to 0.06 mm at the 90th iteration, while the PID and the PD plus PD-type ILC controller lead to an RMSE along the x-axis equal to 1.48 mm and 0.12 mm, respectively. Fig. 6 shows the tracking error of joint 1 and joint 2, respectively, (the tracking error of joint 3 is similar to that of joint 2 due to the nature of the trajectory), for the 1st, 10th, 30th, 50th and the 90th iteration. It is observed that the desired trajectory has been obtained successfully with the increase of the iteration number despite the existence of the model uncertainty and the external disturbances. Fig. 7 represents the torque control of joint 1 and joint 2, respectively. It is shown that the torque profile remains nearly the same from the first iteration to the 90th iteration, which provides an advantage of the proposed controller where the tracking performances are enhanced through the iteration without requiring more control energy.

Case 2: In order to evaluate the ability to track the desired trajectory in the presence of payloads, an additional load of 200 g is introduced on the travelling plate of the Delta robot, from 30 iterations to 90 iterations.

Table 2: Tracking performance through iterations under additional load of 200 g.

Iteration	1	10	30	50	90
RMSE x-axis (mm)	2.17	1.60	2.76	1.22	0.20
MaxAE x-axis (mm)	3.43	2.21	4.02	1.85	0.33
RMSE z-axis (mm)	1.10	0.85	1.30	0.58	0.16
MaxAE z-axis (mm)	2.40	1.73	2.73	1.16	0.33

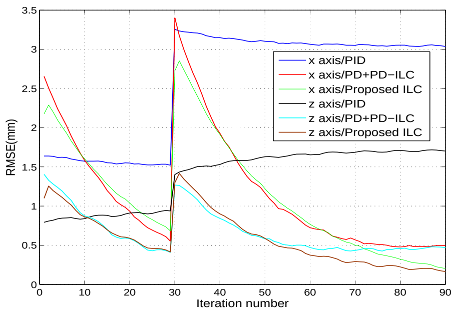


Figure 8: The RMSE along the iteration axis-Case 2.

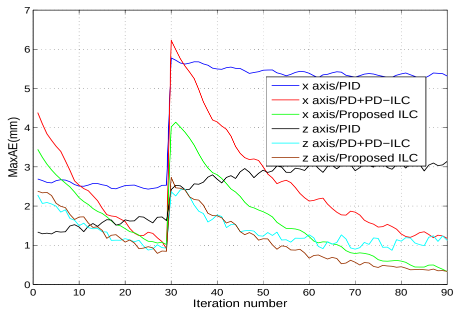
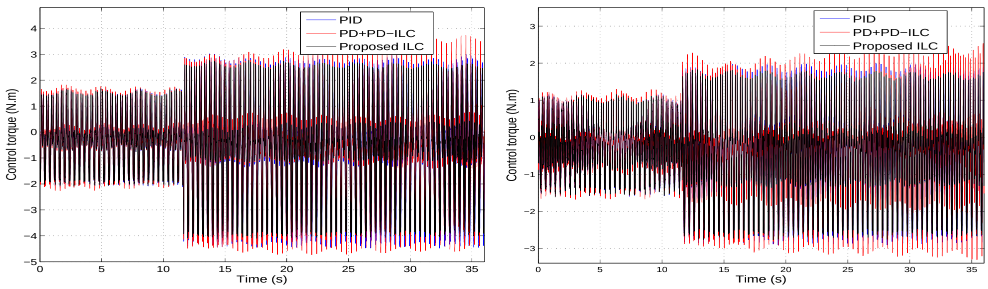


Figure 9: The RMSE along the iteration axis-Case 2.



(a) (b)

Figure 10: The control torque-Case 2. (a) joint 1, (b) joint 2.

The simulation results are presented in Fig. 8 to Fig. 10. It is observed that the PID controller lost its performances after the introduction of the additional load, while the performances of the PD plus PD-type became constant between the 60th and the 70th iterations, then start diverging after that. Meanwhile, the proposed ILC is still the one giving us better performances, where, on one hand, the RMSE and the MaxAE decrease with a rate faster than the traditional ILC and continue to decrease even at the 90th iteration. On the other hand, the control torque of the proposed controller is smaller than the PID and the PD plus PD-type ILC, which provides a significant importance to the proposed approach. Table 2 summarises the tracking performance obtained under an additional load of 200 g.

5 Conclusion

In this work, a model-based iterative learning scheme has been proposed for the trajectory tracking of robot manipulators with model uncertainty and subjected to external disturbances. In order to decrease the coupling effect, a model-based controller has been introduced and combined with an ILC and a robust control term to benefit from the repetition of the task and to reject the model uncertainty and external disturbances. The asymptotic convergence has been demonstrated using the Lyapunov method. It has been shown that the tracking position and velocity errors decrease through the iterations regardless of the influence of the model uncertainty and the external disturbances. Simulation results confirm the feasibility and the effectiveness of the proposed control scheme compared to the PID and the PD plus PD-type ILC. Otherwise, the control energy is still limited and does not increase with the number of iterations.

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