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Capacity in Anisotropic Sobolev Spaces

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Abstract: This paper is devoted to the study of the theory of capacity in an anisotropic Sobolev space $W^{1,\vec{p}}(\Omega)$, where Ω is a bounded set of $\mathbb{R}^N (N \geq 2)$, $\vec{p} = (p_0, p_1, ..., p_N)$ with $1 < p_0, p_1, ..., p_N < \infty$. We will define the $C_{k,\vec{p}}$ capacity and prove its main properties, especially, it will be shown that $C_{k,\vec{p}}$ defines a Choquet capacity. To illustrate our results, we will present an application of this capacity.

Keywords: *anisotropic Sobolev spaces; capacity; potential.*

Mathematics Subject Classification (2010): 31C15.

1 Introduction

The theory of capacity and non-linear potential in the classical Lebesgue space $L^p(\Omega)$ $(1 < p < \infty)$ was studied by Maz'ya and Khavin in [16] and Meyers in [18]. These authors introduced the concept of capacity and non-linear potential in these spaces and provided very rich applications in functional analysis, harmonic analysis, theory of partial differential equations and theory of probabilities.

It has been developed specially by Adams [1], by Hedberg in [13], by Hedberg and Wolff in [14] and others. The Sobolev capacity for constant exponent spaces has found a great number of applications (see [12, 15]) and, for example, Boccardo et al. [8] studied the existence and non existence of solutions of the following problem:

$$(\mathcal{P}) \begin{cases} -\triangle u + u \mid \nabla u \mid^2 = \mu & in \ \Omega, \\ u = 0 & on \ \partial\Omega, \end{cases}$$

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