



Averaging Method and Boundary Value Problems for Systems of Fredholm Integro-Differential Equations

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Abstract: In this paper, an analogue of Bogolyubov's first theorem of the averaging method for systems of Fredholm integro-differential equations is established. The averaging method is also applied to boundary value problems for such systems. It is shown that if a boundary value problem for an averaged system, which is a system of ordinary differential equations, has a solution, then the original problem is solvable as well.

Keywords: *Fredholm integro-differential equation; boundary-value problem; averaging method.*

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1 Introduction

In this paper, we study systems of Fredholm integro-differential equations

$$\frac{dx}{dt} = \dot{x} = \varepsilon X(t, x, \int_0^{\frac{T}{\varepsilon}} \varphi(t, s, x(s)) ds) \quad (1)$$

subject to the Cauchy conditions

$$x(0) = x_0, \quad (1')$$

or to the boundary conditions

$$F(x(0), x(\frac{T}{\varepsilon})) = 0, \quad (1'')$$

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