Nonlinear Dynamics and Systems Theory, **21** (1) (2021) 27–55

Existence and Asymptotic Behavior of Unbounded Positive Solutions of a Nonlinear Degenerate Elliptic Equation

A. Bouzelmate * and A. Gmira

Department of Mathematics, Faculty of Sciences, University Abdelmalek Essaâdi, B.P. 2121, Tetouan, Morocco

Received: July 8, 2020; Revised: December 29, 2020

Abstract: This paper is a contribution to the study of the elliptic equation

$$\Delta_p u + \alpha u + \beta x \cdot \nabla u + |x|^l \, u^q = 0 \quad \text{in } \mathbb{R}^N,$$

where $p > 2$, $q > 1$, $N \geq 1$, $\alpha < 0$, $\beta < 0$ and $l < 0$.
If $q \leq p-1$ or $q > p-1$ and $\dfrac{\alpha}{\beta} \neq \dfrac{l+p}{q+1-p}$ or $\dfrac{\alpha}{\beta} = \dfrac{l+p}{q+1-p} \geq \dfrac{N-p}{p}$, we prove the existence of unbounded radial solutions u and we obtain their asymptotic behavior. In particular, if $\dfrac{\alpha}{\beta} < \dfrac{-l}{q-1}$, $\lim\limits_{r\to+\infty} r^{l/(q-1)} u(r) = \left(\dfrac{\beta l}{q-1} - \alpha\right)^{1/(q-1)}$.

Keywords: *nonlinear parabolic problem; nonlinear degenerate elliptic equation; self-similar solutions; nonlinear dynamical system; unbounded solutions; energy function; asymptotic behavior.*

Mathematics Subject Classification (2010): 34L30, 35J60, 35K92, 35K55.

* Corresponding author: mailto:abouzelmatearij@uae.ac.ma

