



Upper and Lower Solutions for Fractional q -Difference Inclusions

S. Abbas¹, M. Benchohra² and J. R. Graef^{3*}

¹ *Department of Mathematics, Tahar Moulay University of Saïda, P.O. Box 138, EN-Nasr, 20000 Saïda, Algeria.*

² *Laboratory of Mathematics, Djillali Liabes University of Sidi Bel-Abbès, P.O. Box 89, Sidi Bel-Abbès 22000, Algeria, and Department of Mathematics, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia.*

³ *Department of Mathematics, University of Tennessee at Chattanooga, Chattanooga, TN 37403, USA.*

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Abstract: This paper deals with some existence results for a class of boundary value problems for Caputo fractional q -difference inclusions by using set-valued analysis, fixed point theory, and the method of upper and lower solutions.

Keywords: *fractional q -difference inclusion; upper solution; lower solution; boundary condition; fixed point.*

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1 Introduction

Fractional differential equations and inclusions have been applied in various areas of engineering, mathematics, physics, and other applied sciences. Recently, considerable attention has been given to the existence of solutions of initial and boundary value problems for fractional differential equations and inclusions with Caputo fractional derivatives. The method of upper and lower solutions has been successfully applied to study the existence of solutions for differential equations and inclusions; see [1–5, 11, 12] and the references therein.

The study of fractional q -difference equations was initiated early in the 20-th century [6, 14] and has received significant attention in recent years [10, 16]. Some interesting details about initial and boundary value problems for q -difference and fractional

* Corresponding author: <mailto:John-Graef@utc.edu>