



# Asymptotic Behavior in Product and Conjugate Dynamical Systems Using Bi-Shadowing Properties

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**Abstract:** In this paper, we study the persistence of asymptotic behavior of trajectories generated by the product of discrete-time dynamical systems and also generated by conjugate systems as well, using some shadowing and bishadowing properties. In particular, we establish a relationship between the asymptotic behavior of product systems and their subsystems and give some new results in this direction. We also show that bishadowing is invariant for the systems that are topologically conjugate under certain conditions.

**Keywords:** *bishadowing; dynamical systems; product systems; conjugate systems.*

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## 1 Introduction

In recent years, the theory of shadowing has become a significant part of qualitative theory of dynamical systems. It plays an important role in the investigation of stability theory and asymptotic behavior of discrete systems, see also [2, 3]. It is usually used to verify computer calculations of the system by ensuring the existence of the true trajectory of the system close to the calculated trajectory (also known as the pseudo trajectory), see [4, 15]. Shadowing firstly appeared in the work of Anosov [6], see also Bowen [7] and is now developed as part of the global theory of dynamical systems, see Palmer [20] and Pilyugin [21]. The relationship between pseudo trajectories and true trajectories became more important particularly in chaotic systems.

Nevertheless, it is natural to pose the inverse problem: given a dynamical system and a family of approximate trajectories, is it possible, for any true trajectory to find

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