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Solution to the Critical Burgers Equation for Small Data in a Bounded Domain

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Abstract: Solvability of Dirichlet's problem for the subcritical fractional Burgers equation is discussed here in the base spaces $D((-\Delta)^{\frac{s}{2}})$, $s \geq 0$ fixed. A unique solution in the *critical case* ($\alpha = \frac{1}{2}$) for small data is obtained next as a limit of the $X^{\frac{1}{2\alpha}}$ solutions to the subcritical equations, when the exponent α of $(-\Delta)^{\alpha}$ tends to $\frac{1}{2}^{+}$.

Keywords: *fractional Burgers equation; global solvability; critical equation.*

Mathematics Subject Classification (2010): 35S11.

1 Introduction

We consider the Dirichlet boundary value problem for the fractional Burgers equation in a bounded interval $I \subset \mathbb{R}$

$$\begin{aligned} &u_t + \frac{1}{2}\nabla u^2 + (-\Delta)^{\alpha} u = 0, \quad x \in I \subset \mathbb{R}, t > 0, \\ &u = 0 \text{ on } \partial I, \\ &u(0,x) = u_0(x), \end{aligned} \tag{1}$$

where $\alpha \in [\frac{1}{2}, 1]$ is a fractional exponent.

In our work we use the following Balakrishnan's definition of the fractional Laplacian (see [14]):

$$(-\Delta)^{\beta} g = \frac{\sin(\beta\pi)}{\pi} \int_0^{\infty} s^{\beta-1}(sI - \Delta)^{-1}(-\Delta) g \, ds, \quad g \in D(-\Delta), \quad \beta \in (0,1).$$

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